# Scaling Laws of Social-Broadcast Capacity for Mobile Ad Hoc Social Networks

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Abstract—In this paper, we mainly investigate capacity scaling laws of the mobile ad hoc social networks (MAHSNs) where social networking applications are implemented over the underlying mobile ad hoc networks. We model the real-world mobility pattern of mobile social users by introducing a *clustered model* that defines two levels of mobility, i.e., strong mobility and weak mobility, according to the impacts of mobility on the gain of network capacity. To address the formation of social relationships among mobile social users, we adopt a distance and densityaware social model called population-distance-based model that comprehensively and practically takes account of the clustering levels of friendship degree and distribution. Under those models, we derive the capacity scaling laws for social-broadcast sessions in MAHSNs. The results provide the exploratory insights into the impacts of users' mobility patterns and the formation of social relationships on the network capacity of MAHSNs.

*Index Terms*—Mobile Social Networks, Network Capacity, Social-Broadcast.

# I. INTRODUCTION

In recent years we have witnessed a lot of social networking applications based on the PC emerge, such as Facebook, Twitter, Myspace, Skype and so on. The number of social network users is dramatically increasing. According to the report of TechCrunch website on Apr 29, 2014, the Twitter user growth accelerated to 5.8%, with 255 million monthly users and 78% of them were on mobile. Nowadays, as the mobile devices like smartphones and mobile Pads become more and more prevalent, the wireless mobile network is very common and becomes the main communication implementations for real-life networking applications gradually [1]. On account of the popularity of mobile and social applications based on the PC saturate at present, there are an increasing number of applications moving to the mobile client, such as WeChat and so on. Moreover, according to science and technology media uTest's message on March 13th, 2014, the number of users of mobile APPs was greater than that of PC for the first time, and the mobile terminal has become the main internet browsing terminal in America. Obviously, the mobile social networks (MSNs) are becoming more and more popular.

In this work, we focus on mobile social networks under the ad hoc communication architecture, called *mobile ad hoc social networks* (MAHSNs), which have many advantages in terms of base stations communication offloading, users' privacy preservation, and network delay reduction. To improve and evaluate the performance under dedicated protocols for MAHSNs, it is indispensable to investigate fundamental limits of system performance, i.e., the optimal achievable performance. Specifically, we study the network capacity, a basic metric of fundamental limits of performance, for data dissemination in MAHSNs. To examine the network capacity as the network size gets large, we further limit the scope of this work to the issue of capacity scaling laws.

In MAHSNs, the underlying users' social relationships generate depending on social users' mobility patterns, and finally by cooperating with social users' mobility patterns, they exert the influence over the capacity of the coupled network system. Compared with the study of capacity scaling laws for general mobile ad hoc networks (MANETs) [2], the problem for MAHSNs has a particular challenge: How do we analyze the combined impacts of the users' mobility patterns and the underlying social relationships on the network capacity?

To investigate the network capacity of data dissemination in MAHSNs, we first analyze the geographic characteristics of data dissemination sessions in MAHSNs, i.e., the spatial distribution of traffic sessions (the location distribution of sources and destinations). Based on the layered modeling method in [3], we introduce the threelayered model that consists of the physical network layer (Layer 1), social relationship layer (Layer 2), and application session layer (Layer 3), as illustrated in Fig.1. For the purpose of deriving the spacial distribution of traffic sessions depending on users' geographical distribution, we can adopt two steps to clarify the correlation between Layer 3 and Layer 1: Firstly, we start with dredging the correlation between Layer 2 and Layer 1, i.e., the relevance between the formation scheme of users' social relationships and the distribution model of users' geographical locations. Secondly, we model the correlation between Layer 3 and Layer 2, i.e., the relevance between the traffic pattern for a specific social application and the topology of users' social relationship network. Next, we will formulate the corresponding models to clarify these two relevances in the process of modeling Layer 1, Layer 2, and Layer 3, respectively.

**Physical Network Layer:** We introduce the clustered model from [4] to express the clustering phenomenon [5], [6] of real-world mobile users. The work uses two clustering parameters (m(n), r(n)) to denote this model, in which n denotes the total number of users and m(n) and r(n) represent the number and the radius of clusters, respectively. For each user, we propose a

notion of *home-point* according to the frequency of its check-ins. We define a tension coefficient  $\eta(n) = n^{\varpi}$  to express the degree of mobility strength, where  $\varpi \in [0, 1/2]$  is the *tension exponent*. According to different mobilities, we denote critical transmission range  $\tau(n) = \sqrt{\log m(n)/m(n)}$  to divide the mobile nodes into two cases named *strong mobility* (when  $\eta(n) = o(1/\tau(n))$ ) and *weak mobility* (when  $\eta(n) = \omega(1/\tau(n))$ ).

Social Relationship Layer: The significant difference on Layer 1 from the scenario in [3] is that all users are assumed to be static. Obviously, it is impractical for the mobile nodes in MAHSNs. So, the social formation model proposed in [3] cannot apply directly to this study. Using the stability of home-point locations, we bridge the social formation model for static users [3], [7], [8] to the mobile cases in MAHSNs, and propose a modified population-distance-based social formation model due to its reality and practicality. Specifically, we let the static home-points of mobile nodes map into the social relationship layer, and let these homepoints choose their own relevant points randomly and independently. We denote such social formation model by  $\mathcal{P}(\delta, \gamma, \beta)$ , where  $\delta \in [0, \infty)$  represents clustering exponent of node distribution,  $\gamma \in [0,\infty)$  represents clustering exponent of friendship degree, and  $\beta \in [0, \infty)$ represents clustering exponent of friendship formation. Moreover, we validate this social formation model via the check-ins records in Gowalla users' dataset [9].

**Application Session Layer:** We assume that users constantly intend to deliver information to some other users with whom they associate (friends or followers). In this work, we mainly study the typical dissemination session, called social-broadcast, under which the source intends to send information to all of its friends.

Under the three-layered system model, we derive the main results on the social-broadcast capacity for MAHSNs, which can be summarized as follows:

• We obtain the main results of the capacity scaling laws in MAHSNs specifically. To the best of our knowledge, this is the first work to investigate the capacity scaling laws of MAHSNs by taking account of the impacts of social relationships on the traffic session pattern. In this paper, to make the physical network layer of the three-layered social network model more practical, we improve its apparent shortage caused by the impractical assumption that the nodes are static and uniform. We also make research on the relationship between the capacity and the coefficients  $\gamma, \beta$  in the population-based model  $\mathcal{P}(\delta, \gamma, \beta)$ , where  $\delta \in [0, \infty)$  represents clustering exponent of node distribution,  $\gamma \in [0,\infty)$  represents clustering exponent of friendship degree, and  $\beta \in [0, \infty)$ represents clustering exponent of friendship formation [3]. We have a conclusion that larger  $\gamma, \beta$  can lead to bigger capacity.

• We show a special session scheme in our threelayered mobile social network model, named socialbroadcast. It means that a source node communicates with all its friend nodes.

• We introduce the population-based model  $P(\delta, \gamma, \beta)$  to choose the destination nodes more practically. This is





# Fig. 1: Three-Layered Model. [3].

different from the independent and random method in aforementioned work.

The rest of this paper is organized as follows: In Section 2, we introduce the three-layered model and show the system assumptions in every layer. We present the main results on social-broadcast capacity in inhomogeneous MAHSNs in Section 3. In Section 4, according to two different mobility cases, we devise two kinds of social-broadcast scheduling schemes correspondingly. An efficient routing policy for social-broadcast is proposed. In Section 5, we present the specific analysis process of per-node capacity under two different mobility cases. Then we review the literatures and highlight the differences between our work and some related ones in Section 6. Finally, we conclude this paper in Section 7.

#### **II. SYSTEM ASSUMPTIONS AND NOTATIONS**

In this paper, we introduce a new social network model: Three-layered social network model from literature [3]. It presents a three-layered perspective for the mobile ad hoc social networks (MAHSNs), consisting of the physical network layer, social relationship layer, and application session layer, as shown in Fig.1.

## A. Physical Network Layer

In our model, the bottom layer is physical network layer. In this layer we give our mobility model and communication and interference model to analyze the capacity scaling laws in MAHSNs.

1) Mobility Model: In our mobility model, the network area is considered as a torus  $\mathcal{O}$ , with n (a random number) wireless ad hoc nodes moving on its surface non-uniformly. The torus can avoid border effects. For convenience, we let the network area be 1 in this paper.

In the process of studying capacity scaling laws in MAHSNs, we find that the mobility model has a property of the spatial inhomogeneity of nodes density. It can be described as follows: The moving range of a node is not the whole network but a certain region of the network. That is to say, for most of the time, the node moves in a certain small territory. It moves far from this region with low probability or never moves to other territories, which is called the clustering phenomenon in [4]. This phenomenon is evident in MAHSNs particularly. User density is higher in some aggregate regions and lower in some sparse regions.

In social networks, the non-uniform user density in the regions brings the spatial inhomogeneity. In order to describe this phenomenon, we use the clustered model [4]. It is denoted by a two-tuples (m(n), r(n)), where m(n) symbols for the number of clusters and r(n)symbols for the radius of a cluster. And each cluster has a center home-point which is expressed as  $X^h$ . Then we suppose that each mobile node *i* moves into its cluster and has a home-point  $X_i^h$ , which is the position of the maximal active probability for node *i*.

From the statements above, since the clustered model can describe the spatial inhomogeneity property well, we use it to help us study the capacity scaling laws in MAHSNs. We find that besides the spatial inhomogeneity property, another characteristic also affects the capacity scaling laws in MAHSNs, which is the node mobility's degree. The intensity of the nodes' mobility is different: Some mobile nodes can move far from their home-points to transmit messages, but some only can move near around the home-points. Because of this property, it is necessary to divide the mobility into two cases: strong mobility case and weak mobility case.

To characterize these two cases, we give the definitions of two parameters in our work as follows:

• Tension Coefficient: We define the tension coefficient as  $\eta(n) = n^{\varpi}$  and it stands for the degree of mobility strength, where  $\varpi \in [0, 1/2]$  represents *tension* exponent [4]. The tension can be interpreted as the pull of a rubber band that is fixed at the home-point. When the tension coefficient is large, the rubber band's pull is big and so it is hard for the mobile node to move far, and then we say the mobility is weak. On the contrary, when the coefficient is small, the mobility is strong. This parameter represents the node's ability of moving away from the home-point.

• Critical Transmission Range: We define the critical transmission range as  $\tau(n)$ . It stands for the minimal transmission range that would guarantee the network connectivity in the case that nodes still remain at their home-points [4]. According to the clustered model, we have  $\tau(n) = \sqrt{\log(m(n))/m(n)}$ , where m(n) represents the number of clusters.

After giving these two parameters, we use the density function of node *i* around its home-point  $X_i^h$  to get the mobile radius of node i [4]. We describe this density function  $\phi_i(X)$  as below:

$$\phi(X - X_i^h) = \frac{s(\eta(n) ||X - X_i^h||)}{\int_{\mathcal{O}} s(\eta(n) ||X - X_i^h||) dX}, \quad (1)$$

where  $s(\eta(n) || X - X_i^h ||)$  represents a non-increasing continuous function, and  $||X - X_i^h||$  denotes the distance between the node and its home-point. We notice that the order of the denominator of Eq.(1):  $\int_{\mathcal{O}} s(\eta(n) || X -$  $X_i^h ||) dX$  is  $\frac{1}{\eta^2(n)}$ . It tells us the node has a very likely chance to move in an area of  $\Theta\left(\frac{1}{\eta^2(n)}\right)$ , then the mobile radius can be limited to  $\Theta\left(\frac{1}{\eta(n)}\right)$  roughly. We define two kinds of mobility cases through the regulars below: **Regular 1**: When  $\eta(n) = o\left(\frac{1}{\tau(n)}\right)$ , the case is strong

mobility.



Fig. 2: The number of friends. [3].

When the node has strong mobility, it can carry data and move to the destination nodes for exchanging messages directly. And from Eq.(1) in clustered model, we know the mobile radius:  $\Theta\left(\frac{1}{\eta(n)}\right)$ . So in the strong case, to guarantee the network connectivity, we give the critical transmission range as  $\frac{1}{\eta(n)}$  at most, i.e.,  $\tau(n) = o\left(\frac{1}{\eta(n)}\right)$ . Then we obtain Regular 1. We can see that the mobility of nodes is a style of transmitting data. So the mobility plays an important role in exchanging data. Meanwhile this case can take full advantage of mobility and make the capacity as big as possible.

**Regular 2**: When  $\eta(n) = w\left(\frac{1}{\tau(n)}\right)$ , the case is weak mobility.

When the node has weak mobility, because the node can't move far or never moves, the network connectivity mainly relies on the critical transmission range. Thus, in the weak case, only given  $\frac{1}{\eta(n)}$  at least, i.e.,  $\tau(n) =$  $w\left(\frac{1}{\eta(n)}\right)$ , can the critical transmission range guarantee the network connectivity. So we obtain Regular 2. Here the effect of mobility on system performance is weak. The network connectivity mainly depends on the transmission range of each node, not the node mobility.

2) Communication and Interference Model: We introduce the physical model from [10] as our communication and interference model in this paper.

#### B. Social Relationship Layer

The middle layer is social relationship layer. In this layer, according to the population-based model [3], we propose a modified population-distance-based social formation model. Not only it is a distance and density-aware social model, but also it has the mobile nodes. This new model improves other models in previous works, such as the distance-based model in [7] and the rankbased model in [8]. Obviously, this model is more useful, practical and convenient.

From the physical network layer we know that although the nodes are mobile and non-uniform, the homepoint of each node is static. So we let these homepoints map into the social relationship layer. This way is reasonable due to the clustered model. Then each mobile source node  $v_k$  in physical network layer corresponds to a static node in social relationship layer. From the population-based model  $\mathcal{P}(\delta, \gamma, \beta)$ , we know that the number of friends of a node obeys the degree distribution. Furthermore, from the model we can get how those friends are distributed in the network. Usually, the distribution of each node's friends is different. Note that we get some useful system settings from [3]:

• Degree Distribution of Social Relationships

Let  $q_k$  denote the number of friends of node k. From [3], we know  $q_k$  is a variable quantity that obeys the degree distribution. For example, in Fig.2 the number of friends is 5 at a certain probability.

• Distribution of Anchor Points and Friends

We can use Theorem 1 in [3] for every session to get the distribution of anchor points, which is shown in Fig.3. Next, using the nearest-principle position of these anchor points, we obtain the distributions of the  $q_k$  friends as shown in Fig.4.



Fig. 3: The distribution of anchor points. [3] The persons stand for the real mobile nodes, the asterisk  $P_{k_i}$  stands for the anchor points and the center person  $V_k$  stands for the reference node.

# C. Application Session Layer

The upper layer is application session layer. In this layer, different applications determine different social sessions. In [3], some different social sessions were introduced, such as posts on Facebook. In those scenarios, the source node sends messages to all of its friends. This pattern is called social-broadcast. Also the source node can transmit data to one friend or multiple selected friends, which are named social-unicast and social-multicast respectively, such as the WeChat.

In this paper we construct our session as the socialbroadcast based on the population-based model, i.e., one node delivers messages to all its friends. So this session model is different from Fig.1 and is shown in Fig.5.

Next we will describe social-broadcast session in details. Let  $\mathcal{V} = \{v_1, v_2 \cdots v_k\}$  denote a set of communication nodes moving in the network  $\mathcal{O}$ . The corresponding home-point of each node is denoted by a set  $X^h = \{X_1^h, X_2^h \cdots X_k^h\}$ . To express one social-broadcast session in the population-based model, we define the socialbroadcast session by the set  $\mathcal{S}_k := \{v_k\} \cup \{\mathcal{F}_k\}$  [3], where  $v_k$  represents the source node of every element in  $\mathcal{F}_k = \{v_{k_i}\}_{i=1}^{q_k}$ . We use  $v_{k_i}$  to stand for the nearest node to the corresponding anchor point in the social relationship layer. We can see that  $v_{k_i}$  represents the destination node, which also has the home-point correspondingly. And the set  $X_{k_i,F}^h = \{X_{k_1,F}^h, X_{k_2,F}^h \cdots X_{k_{q_k,F}}^h\}$  can express the friends' home-points of the source node  $v_k$ .

From the population-based model, we suppose that one source node  $v_k$  has  $q_k$  friends. So under socialbroadcast the source node  $v_k$  communicates with all its friends, i.e., in every session each source node  $v_k$  has  $q_k$  destinations. The number of friends of each node is a random variable that obeys the probability distribution [3]. This way is different from other works in which the destinations are chosen independently and randomly.



Fig. 4: The distribution of friends. [3] Similarly the node  $V_k$  stands for the reference point, and the asterisk  $P_{k_i}$  stands for the anchor points; We use the rule that chooses the nearest users to the anchor points  $P_{k_i}$  to be the friends, which are denoted by the nodes  $V_{k_i}$ , i.e., the reference point's friends.



Fig. 5: The social-broadcast session. [3] The source node sends message to its *i*-th friend with probability  $p_i$ .

#### D. Network Capacity for Social Sessions

As known to all, the network capacity is a universal concept. In this paper we quote the definition in [4] and use analogous notation. We assume that the rate of packets arriving at every node is  $\lambda$  packets per-slot and the per-node capacity of the system is  $\Theta(h(n))$ . Given a sequence of uniform permutation traffic patterns with rate  $\lambda^{(n)} = h(n)$ , there exist two constants  $c_1, c_2$ , where  $c_1 < c_2$  and both of the following properties hold:

$$\begin{cases} \lim_{n \to \infty} \Pr\{c_1 \lambda^{(n)} \text{ is sustainable}\} = 1\\ \lim_{n \to \infty} \Pr\{c_2 \lambda^{(n)} \text{ is sustainable}\} < 1. \end{cases}$$
(2)

When there are n nodes, we can say that the network capacity can reach  $\Theta(nh(n))$  in this case.

# III. MAIN RESULTS

For convenience, we list some mainly used notations in TABLE I. In this work, we concentrate on studying a special case. We suppose that the home-points are distributed in the area uniformly and independently. As the home-points are uniform, the clustered model would become the uniform model [4]. It has said that in the uniform model m(n) = n is satisfied. So in this special case we derive that  $\tau(n) = \sqrt{\log m(n)/m(n)} = \sqrt{\log n/n}$ . The population-based model  $\mathcal{P}(\delta, \gamma, \beta)$  also tells us that when the nodes are uniform, the clustering exponent of node distribution satisfies  $\delta = 0$ . That is to say, we specifically reduce the complexity from three dimensions  $(\delta, \gamma, \beta) \in [0, \infty)^3$  to two dimensions  $(\gamma, \beta) \in [0, \infty)^2$ .

Under these assumptions, we obtain the bounds on social-broadcast capacity in two different mobility cases in MAHSNs. The results are given as follows.

TABLE I: Main symbols used in this paper					
Symbols	Descriptions				
m(n)	the number of clusters				
r(n)	the radius of a cluster				
ω	tension exponent				
$\eta(n)$	tension coefficient of home-point				
$\tau(n)$	critical transmission range				
$X_i(t)$	the position of node $i$ at time $t$				
$d_{ij}(t)$	the distance between node $i$ and $j$ at time $t$				
$\mathcal{V}$	the set of source nodes				
$v_k$	the source nodes				
$X^h$	the set of source nodes' home-point				
$X_k^h$	the home-point of mobile node $k$				
$q_k$	the number of friend nodes of a source node				
$\mathcal{S}_k$	the set of social-broadcast sessions				
$\mathcal{F}_k$	the set of a source node's friends				
$X^h_{k_i,F}$	the set of friends' home-points				
$\mathcal{P}(\delta,\gamma,eta)$	the population-based model				
δ	clustering exponent of node distribution				
$\gamma$	clustering exponent of friendship degree				
β	clustering exponent of friendship formation				
$A_{tes}$	an arbitrary tessellation element				
$N_h(A_{tes})$	the number of home-points in the tessellation $A_{tes}$				
S(i,j)	the probability link capacity between node				
$\mu$ ( $\lambda$ )	i and $j$ under the scheduling scheme $S$				
$S^s$	scheduling scheme under strong mobility case				
$S^w$	scheduling scheme under weak mobility case				
$\lambda$	per-node multicast capacity				

# A. Strong Mobility Case

This case occurs when  $\eta(n) = o\left(\frac{1}{\tau(n)}\right)$ . To make the lower bound tight, i.e., the lower bound asymptotically approaches the upper bound of social-broadcast capacity,

we let the transmission range satisfy  $R_T = \Theta\left(\frac{1}{\sqrt{n}}\right)$ . In strong mobility case, the per-node capacity on social-broadcast is:  $\lambda = \Theta\left(\frac{n}{\eta(n)H(\gamma,\beta)}\right)$ . Combining Lemma 6 and Lemma 9 in [3], we can

obtain the results in TABLE II.

#### B. Weak Mobility Case

This case occurs when  $\eta(n) = \omega\left(\frac{1}{\tau(n)}\right)$ . From this equation we can see the node's mobility is so weak that we have to choose  $R_T = \Theta\left(\sqrt{\frac{\log m(n)}{m(n)}}\right)$  as the transmission range to guarantee the network's connectivity.

In this special case we have  $R_T = \Theta(\sqrt{\frac{\log n}{n}})$ . In weak mobility case, the per-node capacity on social-broadcast is:  $\lambda = \Omega\left(\frac{1}{H(\gamma,\beta)\sqrt{\frac{\log n}{n}}}\right)$ . Also from Lemma 6 and Lemma 9 described in [3],

we can derive our results in TABLE III.

## C. Intuitions and Analysis of Main Results

Firstly, we compare TABLE II with TABLE III. Obviously, the strong mobility case has the bigger capacity. It is well-known that the mobility can increase capacity [11]. So we validate the correctness of this conclusion from another perspective.

Secondly, we mainly concentrate on the strong mobility case. We analyze the relationship between the capacity and the coefficients  $\gamma$ ,  $\beta$  in the population-based

TABLE II: Social-Broadcast Capacity in Strong Mobility Case

$\gamma$	β	$\lambda$	
	$\beta > 2$	$\Theta\left(\frac{1}{\eta(n)}\right)$	
	$\beta = 2$	$\Theta\left(\frac{1}{\eta(n)\log n}\right)$	
$\gamma > 2$	$1<\beta<2$	$\Theta\left(\frac{n^{\frac{\beta}{2}-1}}{\eta(n)}\right)$	
	$\beta = 1$	$\Theta\left(\frac{\sqrt{\log n}}{\eta(n)\sqrt{n}}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{\eta(n)\sqrt{n}}\right)$	
$\gamma = 2$	$\beta \geq 2$	$\Theta\left(\frac{1}{\eta(n)\log n}\right)$	
	$1<\beta<2$	$\Theta\left(\frac{n^{\frac{\beta}{2}-1}}{\eta(n)}\right)$	
	$\beta = 1$	$\Theta\left(\frac{\sqrt{\log n}}{\eta(n)\sqrt{n}}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{\eta(n)\sqrt{n}}\right)$	
$3/2 < \gamma < 2$	$\beta \geq 2\gamma-2$	$\Theta\left(\frac{n^{\gamma-2}}{\eta(n)}\right)$	
	$\boxed{1 < \beta < 2\gamma - 2}$	$\Theta\left(\frac{n^{\frac{\beta}{2}-1}}{\eta(n)}\right)$	
	$\beta = 1$	$\Theta\left(\frac{\sqrt{\log n}}{\eta(n)\sqrt{n}}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{\eta(n)\sqrt{n}}\right)$	
	$\beta > 1$	$\Theta\left(\frac{1}{\eta(n)\sqrt{n}}\right)$	
$\gamma = 3/2$	$\beta = 1$	$\Theta\left(\frac{1}{\eta(n)\sqrt{n\log n}}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{\eta(n)\log n\sqrt{n}}\right)$	
$1<\gamma<3/2$	$\beta \ge 0$	$\Theta\left(\frac{n^{\gamma-2}}{\eta(n)}\right)$	
$\gamma = 1$	$\beta \ge 0$	$\Theta\left(\frac{\log n}{\eta(n)n}\right)$	
$0\leq \gamma < 1$	$\beta \ge 0$	$\Theta\left(\frac{1}{\eta(n)n}\right)$	

model  $\mathcal{P}(\delta, \gamma, \beta)$ . The results in TABLE II intuitively show that for both  $\gamma$  and  $\beta$  the range of social-broadcast capacity is  $\left[\frac{1}{\eta(n)}, \frac{1}{n\eta(n)}\right]$ . In this range, the capacity is nondecreasing monotonically. We make an intuitive explanation: when the clustering exponent of friendship degree  $\gamma$  is larger, the number of each user's friends can be limited by a smaller upper bound with high probability, and this situation makes the social-broadcast capacity larger. Similarly, when the clustering exponent of friendship formation  $\beta$  is larger, the friends can be closer to users in a large probability. This brings that the total transmission distance of each social-broadcast session reduces possibly, i.e., the nodes can deliver messages directly. It results in a larger social-broadcast capacity finally. Shortly speaking, the larger  $\gamma$  and  $\beta$  are, the larger the social-broadcast capacity is.

Thirdly, as the large clustering exponent of friendship degree  $\gamma$  and clustering exponent of friendship formation  $\beta$  can lead to the large capacity, we can see in the strong mobility case, when  $\gamma > 2$  and  $\beta > 2$ , the capacity has the ability to reach the maximal value  $\Theta(1)$  theoretically.

#### **IV. SOCIAL-BROADCAST POLICY**

Before obtaining the capacity, we must propose the scheduling policy and routing policy.

$\gamma$	β	$\lambda$	
	$\beta > 2$	$\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$	
	$\beta = 2$	$\Theta\left(\frac{1}{\log n\sqrt{n\log n}}\right)$	
$\gamma > 2$	$1<\beta<2$	$\Theta\left(\frac{n^{\frac{\beta}{2}-1}}{\sqrt{n\log n}}\right)$	
	$\beta = 1$	$\Theta\left(\frac{1}{n}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{n\sqrt{\log n}}\right)$	
$\gamma = 2$	$\beta > 2$	$\Theta\left(\frac{1}{\log n\sqrt{n\log n}}\right)$	
	$1<\beta<2$	$\Theta\left(rac{n^{rac{eta}{2}-1}}{\sqrt{n\log n}} ight)$	
	$\beta = 1$	$\Theta\left(\frac{1}{n}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{n\sqrt{\log n}}\right)$	
$3/2 < \gamma < 2$	$\beta>2\gamma-2$	$\Theta\left(\frac{n^{\gamma-5/2}}{\sqrt{\log n}}\right)$	
	$1<\beta<2\gamma-2$	$\Theta\left(\frac{n^{\frac{\beta}{2}-1}}{\sqrt{n\log n}}\right)$	
	$\beta = 1$	$\Theta\left(\frac{1}{n}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{n\sqrt{\log n}}\right)$	
	$\beta > 1$	$\Theta\left(\frac{1}{n\sqrt{\log n}}\right)$	
$\gamma = 3/2$	$\beta = 1$	$\Theta\left(\frac{1}{n\log n}\right)$	
	$0\leq\beta<1$	$\Theta\left(\frac{1}{n\log^{3/2}n}\right)$	
$1 < \gamma < 3/2$	$\beta \ge 0$	$\Theta\left(\frac{n^{\gamma-5/2}}{\sqrt{\log n}}\right)$	
$\gamma = 1$	$\beta \ge 0$	$\Theta\left(rac{1}{n\sqrt{n/\log n}} ight)$	
$0\leq \gamma < 1$	$\beta \ge 0$	$\Theta\left(\frac{1}{n\sqrt{n\log n}}\right)$	

TABLE III: Social-Broadcast Capacity in Weak Mobility Case

According to two different cases, we formulate two different scheduling policies and routing policy R that are similar to [12]. The only difference is the transmission range and the side length. The scheduling policy in strong mobility case  $S^s$ : it is optimal to choose  $R_T = \Theta\left(\frac{1}{\sqrt{n}}\right)$  as the transmission range and the side length is  $\frac{\sqrt{2}}{\eta(n)}$ . The scheduling policy in weak mobility case  $S^w$ : the transmission range is  $\Theta(\tau(n))$  and the side

#### V. SOCIAL-BROADCAST CAPACITY FOR MAHSNS

In this section, we analyze the social-broadcast capacity in two different mobility cases in details.

## A. Analysis in strong mobility case

length is  $\sqrt{(16+\delta)}\tau(n)$ .

From Section 2, the  $\eta(n) = o\left(\frac{1}{\tau(n)}\right)$  is satisfied in strong mobility case. Then we give the upper bound and lower bound of per-node capacity.

1) Upper bound : To obtain the upper bound of social-broadcast capacity in strong mobility case, we adopt the curve method which is similar to the method in [4]. The steps of this method are described as follows:

1. Use the convex, simple, regular, closed *curve*  $\mathcal{L}$  to partition the area into two regions  $I_{\mathcal{L}}$  and  $E_{\mathcal{L}}$ .

2. Figure out the number of social-broadcast flows crossing  $\mathcal{L}$  denoted by  $N_{\mathcal{L}}$ .

3. Calculate the maximum of the entire traffic crossing

 $\mathcal{L} \text{ by } \sum_{i:X_i^h \in I_{\mathcal{L}}} \sum_{j:X_j^h \in E_{\mathcal{L}}} \mu_{ij}.$ 4. Use the inequality  $\lambda \leq \frac{\sum_{i:X_i^h \in I_{\mathcal{L}}} \sum_{j:X_j^h \in E_{\mathcal{L}}} \mu_{ij}}{N_{\mathcal{L}}}$  to get the upper bound of per-node capacity  $\lambda$ .

The following theorem gives the upper bound of pernode capacity in strong mobility case.

**Theorem 1.** For a mobile ad hoc social network  $\mathcal{O}$ consisting of n mobile nodes that are distributed nonuniformly and move in strong mobility case, by the scheduling scheme  $S^s$ , the upper bound of per-node capacity can be achieved of  $\lambda = \Theta\left(\frac{n}{\eta(n)H(\gamma,\beta)}\right)$ .

Proof. In our social-broadcast session, there are many session flows going through the curve. Owing to the social relationship in the population-based model, different source nodes have different friends' distributions, and then different sessions have different lengths and different probabilities going through the curve. Firstly, we choose one social-broadcast session to study, and then we conduct all social-broadcast sessions.

We use  $l_{k,i}$  to represent the side of the *i*th edge of the kth social-broadcast session. The probability of this social-broadcast session  $l_{k,i}$  passing through  $\mathcal{L}$  is the horizontal component, denoted by  $l_{k,i}^{h}$ . Let  $N_{\mathcal{L}}$  denote the number of social-broadcast flows crossing L. Then from the proof of Theorem 3 in [12] we get the following equation:  $N_{\mathcal{L}} = \sum_{k=1}^{n} \sum_{i=1}^{q_k} l_{k,i}^{h}$ , where  $l_{k,i}^{h}$  is only the length of one social-broadcast session and other sessions have different lengths. As we assume that every one node has  $q_k$  friends, that is to say, each node has  $q_k$  sessions. And from [3] we obtain that  $q_k$  is not a constant value but a variate that obeys the following distribution:

$$\Pr(q_k = l) = \begin{cases} \Theta(l^{-\gamma}), & \gamma > 1; \\ \Theta\left(\frac{1}{\log n} \cdot l^{-1}\right), & \gamma = 1; \\ \Theta(n^{\gamma} - 1) \cdot l^{-\gamma}, & 0 \le \gamma \le 1; \end{cases}$$
(3)

where  $\gamma \in [0,\infty)$  represents the clustering exponent of friendship degree.

According to Lemma 6 of [3], the length of all sessions in population-based model is  $\sum_{k=1}^{n} |EMST(\mathcal{S}_k)| = \Omega(H(\gamma, \beta)). \text{ Thus, we have} \\ N_{\mathcal{L}} \ge H(\gamma, \beta). \text{ And by the result in [4], we can get:} \\ \sum_{i:X_i^h \in I_{\mathcal{L}}} \sum_{j:X_j^h \in E_{\mathcal{L}}} \mu_{ij} \le \frac{n\pi c_1^2}{\eta(n)},$ 

The second step is obtained by Proposition 2 in [4].

We adopt the inequality to obtain the upper bound

$$\lambda = \Theta\left(\frac{n\pi c_1^{-}}{\eta(n)H(\gamma,\beta)}\right) = \Theta\left(\frac{n}{\eta(n)H(\gamma,\beta)}\right)$$

where  $\eta(n)$  represents the tension coefficient of homepoint and from Lemma 6 in [3] we can get our results as listed in the tables in Section 3.

2) Lower bound : We employ the method of lattice view in [3].

Using the similar method as in [12], we work out the probability link capacity between node *i* and node j under scheduling scheme  $S^s$ . From [13], we know the probability link capacity is the maximal traffic flow between them. By Theorem 2 in [4], we get:

$$\mu^{S^s}(i,j) = \Theta(g(n)f(\eta(n) \| X_j^h - X_i^h \|)), \quad (4)$$

 $g(n) = \pi R_T^2(n) \eta^2(n) = \pi c_1^2 \frac{\eta^2(n)}{n},$  where

here  $f(||Y||) = \int_{X \in R} s(||X - Y||)s(||X||)dX.$ Then we use the lattice view method [3] described as

Then we use the lattice view method [3] described as follows:

Partition the area into a sequence of regular tessellations and each side length is <sup>c</sup>/<sub>n(n)</sub>.
 Figure out the *probability link capacity* between

2. Figure out the *probability link capacity* between nodes i and j based on Eq. (4).

3. From Lemma 1 of [4] we can get the number of mobile nodes whose home-points fall in  $A_{tes}$  and  $B_{tes}$ , then we can work out the feasible maximal traffic flow by  $\mu^{S^s}(\overline{d}_{A_{tes},B_{tes}}) \cdot \underline{N}_h(A_{tes}) \cdot \underline{N}_h(B_{tes})$ .

4. From Lemma 1, based on the minimal spanning tree of social-broadcast session  $\sum_{k=1}^{n} |EST(\mathcal{S}_k)| = O(H(\gamma, \beta))$  [3], we work out all the social-broadcast flows going through the  $A_{tes}$ .

5. At last, we can use the inequality below to obtain the lower bound of per-node capacity  $\lambda$ .

$$\lambda \geq \frac{\mu^{S^s}(\overline{d}_{A_{tes},B_{tes}}) \cdot \underline{N}_h(A_{tes}) \cdot \underline{N}_h(B_{tes})}{\mathbf{Pr}^{all}}.$$

**Lemma 1.** In the strong mobility case, the number of social-broadcast flows going through a given tessellation  $A_{tes}$  is  $\min\left(\frac{\sqrt{2}cH(\gamma,\beta)}{\eta(n)} + \frac{Q(\gamma)c^2}{\eta^2(n)}, 1\right)$ .

*Proof.* Firstly, we partition the area into a sequence of regular tessellations, and then assume that there are many social-broadcast flows through the network. Because in the population-based model, the number of friends and their distributions are variables that obey the power law, different session flows have different lengths and the probability of each social-broadcast flow going through a tessellation  $A_{tes}$  is different.

We choose one social-broadcast flow among them to analyze and suppose the length is l, and then let this flow map to the horizontal and vertical projects. They are defined as  $l_h$  and  $l_v$  respectively. In strong mobility case, the side length of  $A_{tes}$  is  $\frac{c}{\eta(n)}$ , where c is a constant. Then we can derive the probability of this social-broadcast flow going through  $A_{tes}$  by the following equation and express it as

$$\Pr(l, A_{tes}) = \frac{c^2}{\eta^2(n)} \left( \frac{l_h + l_v}{\frac{c}{\eta(n)}} + 1 \right) \le \frac{\sqrt{2}lc}{\eta(n)} + \frac{2q_k c^2}{\eta^2(n)}.$$

To get a tight upper bound, the last step employs the law of large numbers. And the parameter n in this law changes to  $q_k$  since this session flow has  $q_k$  destinations.

Note that there are *n* social-broadcast flows and different flows have different lengths and different probabilities to go through this tessellation  $A_{tes}$ . To obtain how many social-broadcast flows go through  $A_{tes}$ , we introduce the following factors.

Because in the population-based model it is not certain whether the length of other social-broadcast flows also equals l, we use Euclidean Spanning Tree (ES-T) to express the whole length of all social-broadcast flows. By Lemma 9 of [3], the length of EST is  $\sum_{k=1}^{n} |EST(\mathcal{S}_k)| = O(H(\gamma, \beta)).$  Note that each node has  $q_k$  friends, so we can get the order of all nodes' friends from the proof of Lemma 6 in [3] by  $\sum_{k=1}^{n} q_k = Q(\gamma)$  where:

$$Q(\gamma) = \begin{cases} \Theta(n), & \gamma > 2; \\ \Theta(n \log n), & \gamma = 2; \\ \Theta(n^{3-\gamma}), & 1 < \gamma < 2; \\ \Theta(n^2/\log n), & \gamma = 1; \\ \Theta(n^2), & 0 \le \gamma < 1; \end{cases}$$
(5)

Therefore, we can obtain the number of socialbroadcast session flows going through the network:

$$\mathbf{Pr}^{all} \le \frac{\sqrt{2}c\sum_{k=1}^{n} |EST(\mathcal{S}_k)|}{\eta(n)} + \frac{\sum_{k=1}^{n} q_k c^2}{\eta^2(n)}.$$

So in the social-broadcast session, we have:

$$\Pr^{all} = \min\left(\frac{\sqrt{2}cH(\gamma,\beta)}{\eta(n)} + \frac{Q(\gamma)c^2}{\eta^2(n)}, 1\right).$$

To figure out the per-node capacity  $\lambda$ , we use routing policy R and scheduling scheme  $S^s$  to gain a lower bound. We assume that there exists a social-broadcast session  $S_1 := \{v_1\} \cup \{\mathcal{F}_1\}$  and adopt routing policy R to construct a social-broadcast tree in virtue of their home-points. In the process of actual transmission, the associated nodes can transmit data when adjacent tessellations' home-points use scheduling scheme  $S^s$ .

From above we can derive the results in Theorem 2.

**Theorem 2.** For a mobile ad hoc social network  $\mathcal{O}$  consisting of *n* mobile nodes that are distributed nonuniformly and move in strong mobility case, by the scheduling scheme  $S^{s}$ , the lower bound of per-node capacity can be achieved of  $\lambda = \Omega\left(\frac{n}{H(\gamma,\beta)\eta(n) + Q(\gamma)}\right)$ .

*Proof.* Assuming that there are two adjacent tessellations:  $A_{tes}$  and  $B_{tes}$ . By Lemma 1 in [4] we can get the lower bound of the number of mobile nodes whose home-points fall in  $A_{tes}$  and  $B_{tes}$  as

$$\underline{N}_h(A_{tes}) = \underline{N}_h(B_{tes}) = \frac{n|A_{tes}|}{2} = \frac{c^2n}{2\eta^2(n)}.$$

The distance between these two tessellations is denoted by  $d_{A_{tes},B_{tes}}$ . Two adjacent tessellations can transmit at the same time without interference. When  $d_{A_{tes},B_{tes}} = \frac{\sqrt{5c}}{\eta(n)}$ , according to Eq.(4), we obtain that  $\mu^{S^s}(\overline{d}_{A_{tes},B_{tes}}) = g(n)f(\sqrt{5c})$ , where the constant c is chosen such that  $f(\sqrt{5c}) > 0$ . According to the step 5, we have the inequality

$$\lambda \ge \frac{\mu^{S^s}(\overline{d}_{A_{tes},B_{tes}}) \cdot \underline{N}_h(A_{tes}) \cdot \underline{N}_h(B_{tes})}{\mathbf{Pr}^{all}}.$$

The numerator means the feasible maximal traffic flow between adjacent tessellations and its value can be obtained from equations above. As for the denominator, we can get its value by Lemma 1.

At the end, we can obtain the lower bound for the strong mobility case:  $\lambda \geq \frac{\pi c^4 c_1^2 \eta(\sqrt{5}c)n}{4\sqrt{2}H(\gamma,\beta)\eta(n) + Q(\gamma)c}$ . It means that the per-node capacity is:  $\lambda = \Omega\left(\frac{n}{H(\gamma,\beta)\eta(n) + Q(\gamma)}\right).$ 

γ	β	$H(\gamma,\beta)\eta(n)$	$Q(\gamma)$
$\gamma > 2$	$\beta > 2$	$n\eta(n)$	
	$\beta = 2$	$n\eta(n)\log n$	
	$1<\beta<2$	$n^{2-eta/2}\eta(n)$	n
	$\beta = 1$	$\frac{n^{3/2}\eta(n)}{\sqrt{\log n}}$	
	$0\leq\beta<1$	$n^{3/2}\eta(n)$	
$\gamma = 2$	$\beta \geq 2$	$n\eta(n)\log n$	
	$1<\beta<2$	$n^{2-eta/2}\eta(n)$	
	$\beta = 1$	$\frac{n^{3/2}\eta(n)}{\sqrt{\log n}}$	$n\log n$
	$0\leq\beta<1$	$n^{3/2}\eta(n)$	
	$\beta \geq 2\gamma-2$	$\eta(n)n^{3-\gamma}$	
$3/2 < \gamma < 2$	$1<\beta<2\gamma-2$	$n^{2-eta/2}\eta(n)$	
	$\beta = 1$	$\frac{n^{3/2}\eta(n)}{\sqrt{\log n}}$	
	$0\leq\beta<1$	$n^{3/2}\eta(n)$	$n^{3-\gamma}$
$\gamma = 3/2$	$\beta > 1$	$n^{3/2}\eta(n)$	
	$\beta = 1$	$n^{3/2}\eta(n)\sqrt{\log n}$	
	$0\leq\beta<1$	$n^{3/2}\log n\eta(n)$	
$1 < \gamma < 3/2$	$\beta \ge 0$	$\eta(n)n^{3-\gamma}$	
$\gamma = 1$	$\beta \ge 0$	$\frac{\eta(n)n^2}{\log n}$	$\frac{n^2}{\log n}$
$0 \le \gamma < 1$	$\beta \ge 0$	$n^2\eta(n)$	$n^2$

TABLE IV: Comparison between Upper and Lower Bounds

3) The comparison of upper bound and lower bound : From Theorem 1 and Theorem 2 we see that the difference between per-node capacity results of the upper bound and lower bound is that the denominator of the lower bound has an extra factor  $Q(\gamma)$ . Since we have known the degree distribution function of  $q_k$  from Eq.(3), we can get its order from Eq.(5). Here we mainly consider the order of the two factors:  $H(\gamma, \beta)\eta(n)$  and  $Q(\gamma)$ . Combining Lemma 6 in [3] and Eq.(5), we give the comparison between the two factors in TABLE IV.

From the comparison, we can derive that  $\Theta(H(\gamma,\beta)\eta(n)) = \Theta(H(\gamma,\beta)\eta(n) + Q(\gamma))$ , which tells us that the upper bound and lower bound are of the same order. So this reflects that our results have a tight lower bound. In other words, there is no gap in our results between the upper bound and lower bound. Then we have the general results given in Theorem 3.

**Theorem 3.** For a mobile ad hoc social network  $\mathcal{O}$  consisting of n mobile nodes that are distributed nonuniformly and move in strong mobility case, by the scheduling scheme  $S^s$ , the bound of per-node capacity can be achieved of  $\lambda = \Theta\left(\frac{n}{\eta(n)H(\gamma,\beta)}\right)$ .

#### B. Analysis in weak mobility case

From Section 2 we have already known that  $\eta(n) = \omega\left(\frac{1}{\tau(n)}\right)$  is satisfied in weak mobility case. In this case we only analyze the lower bound, so we

In this case we only analyze the lower bound, so we directly apply the same method to the lower bound as in strong mobility case. Note that in weak mobility case, the side length is  $\sqrt{(16+\delta)}\tau(n)$  which is different from the strong mobility case.

**Lemma 2.** In the weak mobility case, the number of social-broadcast flows going through a given tessellation  $A_{tes}$  is  $\min\left(\sqrt{2}H(\gamma,\beta)\sqrt{(16+\delta)}\tau(n) + Q(\gamma)(16+\delta)\tau^2(n),1\right)$ .

Here we omit the proof due to the limited space. And the method is similar to that of Lemma 1.

**Theorem 4.** For a mobile ad hoc social network  $\mathcal{O}$  consisting of n mobile nodes that are distributed nonuniformly and move in weak mobility case, by the scheduling scheme  $S^w$ , the lower bound of per-node capacity can be achieved of  $\lambda = \Omega\left(\frac{1}{H(\gamma,\beta)\sqrt{\frac{\log n}{n}}}\right)$ .

*Proof.* Similarly, we suppose that there are two adjacent tessellations  $A_{tes}$  and  $B_{tes}$ . Because of  $\eta(n) = \omega\left(\frac{1}{\tau(n)}\right)$ , this case adopts the scheduling scheme  $S^w$  which uses  $K^2$ -TDMA [14].

By Lemma 2, we get the maximal load, i.e., the maximal number of social-broadcast flows:  $O(\sqrt{2}H(\gamma,\beta)\sqrt{(16+\delta)}\tau(n) + Q(\gamma)(16+\delta)\tau^2(n)).$ 

Thus, we have the lower bound of weak mobility case  $\lambda \geq \frac{1}{\sqrt{2}H(\gamma,\beta)\sqrt{(16+\delta)}\tau(n)+Q(\gamma)(16+\delta)\tau^2(n)}.$  Hence, we obtain  $\lambda = \Omega\left(\frac{1}{H(\gamma,\beta)\sqrt{\frac{\log n}{n}}+Q(\gamma)\frac{\log n}{n}}\right).$  We employ Lemma 9 in [3] and use  $Q(\gamma)$  described in Eq.(5) to analyze this result. Next, we compare the order of the two factor  $H(\gamma,\beta)\sqrt{\frac{\log n}{n}}$  and  $Q(\gamma)\frac{\log n}{n}$ . Using the same method in TABLE IV, we draw a conclusion that the order of the former is bigger than that of the latter. So we have

$$\Theta\left(H(\gamma,\beta)\sqrt{\frac{\log n}{n}} + Q(\gamma)\frac{\log n}{n}\right) = \Theta\left(H(\gamma,\beta)\sqrt{\frac{\log n}{n}}\right).$$
VI. Related Work

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In terms of network capacity, the ground-breaking study was Kumar and Gupta's research [10]. After that in order to improve the network capacity, many works put forward different methods by introducing some special characteristics into network. For example, D.Tse et al. [11] made a great progress on the capacity by considering the mobility. He proposed the store-carry-forward relaying scheme to make the per-node capacity sustain  $\Theta(1)$ .

Meanwhile, people studied different mobility models. The literature [15] carried on an investigation on mobility models in the mobile ad hoc network (MANET) and emphasized its importance. The simplest model was the i.i.d. model which was a random and independent model. Clearly it was an ideal model. After that, some realistic mobility models were proposed, such as random walk mobility model [16], Brownian mobility model [17], and random way-point mobility model [18]. However, in these models the nodes moved globally all over the network area, which was impractical. Then M.Garetto et al. [4] proposed a more practical model: cluster mobility model. Later, Li et al. [12] introduced this cluster mobility model and obtained the per-node capacity on the multicast session in the mobile ad hoc network instead of the unicast session in [4].

The above studies did not consider the effect of social interactions among nodes. The social network was first studied by Milgrams' experiments [19]. Later, Mascolo et al. [20] proposed a new mobility model founded on social network theory. At the same time, Kleinberg [7] proposed the distance-based social model. Later, Nowell et al. [8] indicated that this model underestimated the inhomogeneity of users' geographical distribution and put forward the rank-based model. Recently, Wang et al. [3] proposed a three-layered social network model and also worked out a new source-destination pairs called population-based model which took both the distance and density into account. Then he addressed the issue of capacity scaling laws.

There were many studies on the capacity of the mobile network, as well as that of the social network, like [21]. But few research focused on the capacity scaling laws of *mobile ad hoc social networks* (MAHSNs). Our work is different from other works in that we choose destinations by population-based model instead of i.i.d model, though we introduce the clustered model. Moreover, although we employ the three-layered social network model from [3], we improve its static underlying networks to mobile networks. In all, our study gives a reasonable and practical model of the mobile nodes' relationship and formulates the corresponding traffic session pattern in MAHSNs.

#### VII. CONCLUSION AND FUTURE WORK

In this paper, we mainly address the capacity scaling laws of the MAHSNs under the social-based session formation. We develop the system model in this paper by introducing the three-layered social network model. We use the clustered model with clustering parameters (m(n), r(n)) to characterize the spatial inhomogeneities of nodes density. Then by the population-based model, we obtain the degree distribution of mobile nodes' friends. Finally, we work out the capacity scaling laws of social broadcast. This work can act as the first step of investigating the capacity under the three-layered model with mobile and non-uniform nodes.

There are still several problems to be studied. Firstly, our work only analyzes a special case: the home-points are uniform. Next we will research the non-uniform case, i.e.,  $\delta \neq 0$ . Secondly, we assume that the nodes move in the ad hoc network, which is obviously impractical because in real life the infrastructure is important. So we will study the MAHSNs with infrastructure in our future work. Thirdly, we could further study the capacity scaling laws of other session models, such as social-unicast/social-multicast and social-anycast/social-manycast.

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#### REFERENCES

- P. Agrawal and C. J. Sreenan, "Get wireless: a mobile technology spectrum," *IT professional*, vol. 1, no. 4, pp. 18–23, 1999.
- [2] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.
- [3] C. Wang, L. Shao, Z. Li, L. Yang, X. Li, and C. Jiang, "Capacity scaling of wireless social networks," *IEEE Transactions on Parallel and Distributed Systems(TPDS)*, 2014, doi:10.1109/TPDS.2014.2333524.
- [4] M. Garetto, P. Giaccone, and E. Leonardi, "Capacity scaling in delay tolerant networks with heterogeneous mobile nodes," in *Proc. ACM MobiHoc* 2007.
- [5] C. Cheng, H. Yang, I. King, and M. R. Lyu, "Fused matrix factorization with geographical and social influence in locationbased social networks," in *in Proc. AAAI 2012.*
- [6] M. Lichman and P. Smyth, "Modeling human location data with mixtures of kernel densities," in *in Proc. ACM SIGKDD 2014.*
- [7] J. Kleinberg, "Navigation in a small world," *Nature*, vol. 406, no. 6798, pp. 845–845, 2000.
- [8] D. Liben-Nowell, J. Novak, R. Kumar, P. Raghavan, and A. Tomkins, "Geographic routing in social networks," *PNAS of the United States of America*, vol. 102, no. 33, pp. 11 623–11 628, 2005.
- [9] E. Cho, S. A. Myers, and J. Leskovec, "Friendship and mobility: Friendship and mobility: User movement in location-based social networks," in *Proc. ACM SIGKDD 2011*.
- [10] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.
- [11] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. on Networking*, vol. 10, no. 4, pp. 477–486, 2002.
- [12] L. Zhong, W. Cheng, J. Changjun, and L. Xiangyang, "Multicast capacity scaling for inhomogeneous mobile ad hoc networks," *Ad Hoc Networks*, vol. 11, no. 1, pp. 29–38, 2013.
- [13] W. Huang, X. Wang, and Q. Zhang, "Capacity scaling in mobile wireless ad hoc network with infrastructure support," in *Proc. IEEE ICDCS 2010.*
- [14] C. Wang, X.-Y. Li, C. Jiang, S. Tang, and Y. Liu, "Scaling laws on multicast capacity of large scale wireless networks," in *Proc. IEEE INFOCOM* 2009.
- [15] F. Bai and A. Helmy, "A survey of mobility models," Wireless Adhoc Networks. University of Southern California, USA, vol. 206, 2004.
- [16] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *Proc. IEEE INFOCOM 2004*.
- [17] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Degenerate delay-capacity tradeoffs in ad-hoc networks with brownian mobility," *IEEE/ACM Transactions on Networking (TON)*, vol. 14, no. SI, pp. 2777–2784, 2006.
- [18] G. Sharma and R. Mazumdar, "Scaling laws for capacity and delay in wireless ad hoc networks with random mobility," in *Proc. IEEE ICC 2004.*
- [19] S. Milgram, "The small world problem," *Psychology Today*, vol. 2, pp. 60–67, 1967.
- [20] M. Musolesi and C. Mascolo, "A community based mobility model for ad hoc network research," in *Proceedings of the 2nd international workshop on Multi-hop ad hoc networks: from theory to reality.* ACM, 2006, pp. 31–38.
- [21] L. Fu, J. Zhang, and X. Wang, "Evolution-cast: Temporal evolution in wireless social networks and its impact on capacity," *IEEE Transactions on Parallel and Distributed Systems*, vol. 25, no. 10, pp. 2583–2594, 2014.