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## Multicast capacity scaling for inhomogeneous mobile ad hoc networks

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## ARTICLE INFO

## Article history:

Received 27 January 2012

Received in revised form 4 March 2012

Accepted 12 April 2012

Available online 8 May 2012

## Keywords:

Inhomogeneous mobile network

Multicast capacity

Activity exponent

Clustering parameters

## ABSTRACT

We study multicast capacity for a large-scale spatial inhomogeneous mobile network consisting of  $n$  ad hoc nodes. Under our mobility model, the stationary spatial distribution of a node is non-uniform; each node spends most of the time in a certain region, and rarely (or never) visits out of such region. To characterize the inhomogeneity of the mobility model, we define an *activity exponent*  $\gamma$  and two *clustering parameters*  $(m(n), r(n))$ , where  $\gamma \in [0, 1]$  measures the strength of node mobility,  $m(n)$  denotes the number of clusters,  $r(n)$  denotes the radius of the cluster. We classify the mobility into two cases according to the strength of mobility of each node, called *strong* and *weak* mobility, respectively. Two corresponding scheduling schemes and routing policies combined with the Manhattan multicast tree method are proposed. Suppose there are  $n_s = \Theta(n)$  multicast sessions. Each source has  $n_d$  destinations which are selected randomly and independently. We show that under strong mobility case, the per-node multicast capacity is  $\left(\frac{1}{\sqrt{n_d}^\theta(n)}\right)$  with  $\theta(n) = n^{\frac{1-\gamma}{2}}$ ; under weak mobility case, when  $n_d = O\left(\frac{m(n)}{\log m(n)}\right)$ , the multicast throughput is  $\left(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}}\right)$ ; when  $n_d = \left(\frac{m(n)}{\log m(n)}\right)$ , the multicast throughput is  $\left(\frac{1}{n}\right)$ . Particularly, as a special case, i.e., by letting  $n_d = 1$ , our results unify the previous unicast capacity bounds.

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## 1. Introduction

The problem of asymptotic capacity analysis of wireless network has evoked much interest and many related researches have been done recently. In their pioneering work, Gupta and Kumar [1] defined two interference models, i.e., protocol model and physical model, which are widely used in the follow-up works in this issue. They considered two types of scaling network models, i.e., arbitrary network and random network, where the nodes are arbitrarily and randomly deployed, and derived the unicast capacities under the protocol and physical models, respectively. Thereafter, some works focus on new traffic sessions rather than unicast, such as broadcast [2,3], multicast [4,5], data gathering, i.e., some-to-some communication para-

digm [6]. Furthermore, based on the above static interference models and traffic sessions, other works aim to make the model more realistic or generalized. Some improve the network capacity by introducing a new interference/communication model called generalized physical model [7–11], some are for the capacity of wireless networks using infrastructures and so on [12–14].

In the mobile scenario, the mobility leads to frequently changes of topology, and then brings challenges in the communication protocol design. On the other side of the coin, the mobility can also bring opportunities in improving the network performance. Grossglauser and Tse [15] showed that mobility can increase capacity due to more diversity gain. Under a two-hop store-carry-forward scheme,  $\Theta(n)$  concurrent transmissions can be performed, which achieves per-node throughput  $\Theta(1)$ . Note that only a simple mobility model, i.e., i.i.d model, was considered. More realistic mobility models were studied, such as random walk mobility model [16], Brownian mobility model

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[17], and random way-point mobility model [18,19]. All these models mentioned above are based on a common assumption that all nodes can move globally over the deployment region with a non-infinitesimal probability. But large numbers of real application scenarios [20,21] show that node itself likes to spend most of the time in a certain region and seldom visits other regions. This can also be called restricted/local mobility.

Besides restricted mobility, in the long-term tracing experiments [22,23], another phenomenon is also found. That is many nodes like to aggregate to form a cluster. The entire density distribution is nonuniform, which reflects the properties of nodes group.

From above literature review, we can see the mobility model can be extended to more generalized and realistic. By using proper scheduling and routing schemes, optimal bounds could be achieved. Garetto et al. [24] laid a good basis of analyzing the network capacity under such type of more realistic mobility models. Huang and Wang [25] extended this work into the model with infrastructure support. Both works focus on unicast sessions.

As is well-known, multicast can be regarded as a generalized dissemination session, which can unify both unicast and broadcast. Due to this generality, the capacity analysis and scheme design for multicast session are more challenging than unicast one. In this paper, we study the multicast capacity for mobile networks with a local mobility model. Under this model, an activity coefficient  $\theta(n)$  is defined to represent the degree of mobility strength. The stronger the strength is, the weaker the mobility is. We use two clustering parameters  $(m(n), r(n))$  to characterize the real distribution properties of some nodes aggregating phenomenon, where  $m(n)$  is the number of clusters,  $r(n)$  is the radius of the cluster.

Our major contributions are three folds.

- Through defining activity coefficient, we analyze the relationship between the activity coefficient and critical transmission range. Then, we give a classification of mobility: strong mobility case and weak mobility case.
- Under strong mobility case, the multicast capacity result is  $\left(\frac{1}{\sqrt{n_d}\theta(n)}\right)$ . It implies that mobility plays a fundamental role in communication between different clusters. Mobility increasing capacity is well presented.
- Under weak mobility case, when  $n_d = O\left(\frac{m(n)}{\log m(n)}\right)$ , the multicast throughput is  $\left(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}}\right)$ ; when  $n_d = \left(\frac{m(n)}{\log m(n)}\right)$ , the multicast throughput is  $\left(\frac{1}{n}\right)$ . That shows we cannot look forward mobility to helping us increase capacity.

The rest of the paper is organized as follows. In Section 2, we introduce our system model and put importance on describing the mobility model. Section 3 gives main results about multicast capacity in inhomogeneous mobile ad hoc networks. In Section 4, two different mobility cases are defined, correspondingly, two kinds of multicast scheduling schemes are proposed. In Section 5, we present an efficient routing policy for studying the bounds on multicast capacity under two mobility cases. We review the lit-

erature and highlight the differences between our work and some related ones in Section 6. Finally, we conclude the paper in Section 7.

## 2. System model

### 2.1. Network model

We consider the shape of the network is a torus  $\mathcal{O}$ , with  $n$  nodes moving on its surface, i.e., ignoring the edge effect. The position of mobile node  $i$  at time  $t$  is denoted as  $X_i(t)$ ,  $1 \leq i \leq n$ ; The distance between node  $i$  and  $j$  at time  $t$  is defined as  $d_{ij}(t) = \|X_i - X_j\|$ . In this paper, we normalize the network area to 1 for convenience.

### 2.2. Mobility model

The mobility model has the properties of inhomogeneity which attribute to two elements. One is the different degrees of node mobility, the other is the spatial inhomogeneities of nodes density over the network.

Firstly, in this paper, we consider the mobility pattern is partial, contrasting to the full mobility. That is to say, a node spends most of the time in a local region of the network area, and the probability of a node moving far from the region is relatively low. Assuming that each node  $i$  has a home-point, denoted as  $X_i^h$ , which is located in the center of the small region.  $X_i^h$  is the position of the maximal active probability for node  $i$ .

We define an *activity exponent*  $\gamma$ ,  $\gamma \in [0, 1]$ , then, having an *activity coefficient*  $\theta(n) = n^{-\frac{1-\gamma}{2}}$ . Because nodes move around their home-points according to independent stationary and ergodic processes, we characterize the density function of node  $i$  around  $X_i^h$  by a function  $\phi_i(X)$

$$\phi_i(X) = \phi(X - X_i^h) = \frac{s(\theta(n) \|X - X_i^h\|)}{\int_{\mathcal{O}} s(\theta(n) \|X - X_i^h\|) dX}, \quad (1)$$

where  $s(\theta(n)d)$  is a non-increasing continuous function [24],  $d$  denotes the distance from its home-point.

From Eq. (1), we can see, through imposing on distance  $d$ , the activity coefficient reflects different limitation degrees of node mobility. The greater the value  $\gamma$  is, the stronger the mobile activity is.

Secondly, under the mobility models adopted in the literature, such as i.i.d. mobility model [26–28], random walk mobility model [16], Brownian mobility model [17], random way-point mobility model [18,19], and other mobility models, the distribution of nodes is often assumed to be homogeneous, which conflicts with the realistic mobile world. With a large number of participants and long-term observations, clustering phenomenon has been found [24], that is, some participants aggregate in some regions. User density in these clustering regions is much higher.

Spatial inhomogeneities come by the non-uniform user density in the territory. In order to describe this phenomenon, we define a *clustered model*, denoted by a two-tuples  $(m(n), r(n))$  [24],  $m(n)$  is the number of clusters and  $r(n)$  is the radius of a cluster. Note that we suppose every node has a home-point in the previous part, thus the process

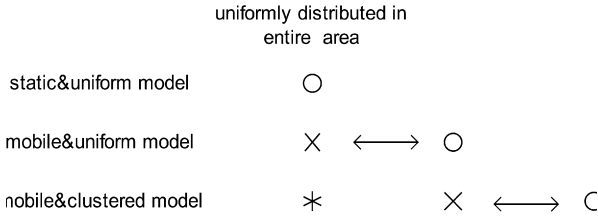


Fig. 1. The evolving course of different model constructions.

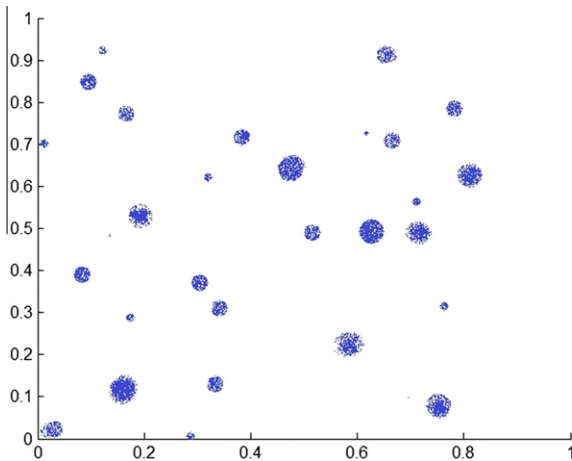
of constructing the model is shown as follows. First, we cast  $m(n)$  cluster centers to the network area uniformly and independently with the radius  $r(n)$ . Then,  $n$  home-points are randomly assigned to these clusters uniformly. Last, each node is associated to its corresponding home-point and belonged to the different cluster.

From the process, we can find the evolving course of different model construction and corresponding distribution state, which is illustrated in Fig. 1 We use marks ‘\*’, ‘×’, ‘o’, ‘←→’ to represent clustered centers, home-points, nodes and the relevancy respectively. Each row represents the construction process of one kind of models. The first column shows a common property of distribution, although the object is different.

The two-tuples satisfies the following conditions so as to make the clustered model more rational.

- (i)  $m(n) = \Theta(n^v), v \in [0, 1]$  and  $r(n) = \Theta(n^{-q}), q \in [0, \infty)$ .
- (ii)  $\lim_{n \rightarrow 0} m(n)r^2(n) = 0$ .
- (iii)  $v - 2q < 0$  and  $0 < q \leq \frac{1-\gamma}{2}$ .

The first condition is intuitive. The second expresses the scenario that the ratio of all clusters to entire area is small. The third guarantees that clusters will not easily overlap with high probability and should not shrink as  $n$  grows [25]. Especially, when  $m(n) = n$ , it is a special case, appearing that all nodes are uniformly distributed. See Fig. 2, a contrast between the clustered model and its special case.



### 2.3. Communication model

We employ the well known physical model [1] as our communication model. Let  $\{X_k(t); k \in \mathcal{T}\}$  be the subset of mobile nodes simultaneously transmitting at time  $t$ . All nodes use a common transmission power level  $P$ . A successful transmission will take place from node  $i$  to  $j$  at time  $t$  only if:

$$\text{SINR} = \frac{P}{N_0 + \sum_{k \in \mathcal{T}, k \neq i} \frac{P}{\|X_k(t) - X_j(t)\|^\alpha}} \beta, \tag{2}$$

where  $N_0$  is an ambient noise power at the receiver,  $\alpha > 2$  is a factor of signal attenuation, and  $\beta$  is a minimum value of signal-to-interference needed by a successful reception at destination node  $j$ .

### 2.4. Construction of multicast

In this paper, assuming that  $\mathcal{V} = \{1, 2, \dots, n\}$  is a set of communication nodes moving in the network  $\mathcal{O}$ . To each node, the corresponding home-point is denoted by a set of  $\{X_1^h, X_2^h, \dots, X_n^h\}$ . Each node  $v_i$  could serve as either source or destination in different multicast sessions. Among the set  $\mathcal{V}$ , we independently and randomly select  $n_s$  nodes as sources,  $n_s = \Theta(n)$ , and each source has  $n_d$  destinations, i.e., there are  $n_s$  multicast sessions. We use  $i \rightarrow \mathcal{V}_{i,D}$  to express one multicast session, where  $v_i$  is one of source nodes and  $\mathcal{V}_{i,D}$  is a set of destination nodes for  $v_i$ ,  $i \in n_s$ . Correspondingly, for each  $v_i$ 's home-point  $X_i^h$ , we have  $n_d$  home-points as one session's destinations. The  $n_d$  home-points constitute a destination set denoted by  $\mathcal{X}_{i,D}^h = \{X_{i1}^h, X_{i2}^h, \dots, X_{in_d}^h\}$ .

### 2.5. Definition of capacity

We suppose packets arrive at each node with rate  $\lambda$  packets per slot. The network is stable if and only if there exists a scheduling scheme which can guarantee the queue in each node does not increase to infinity as time goes to

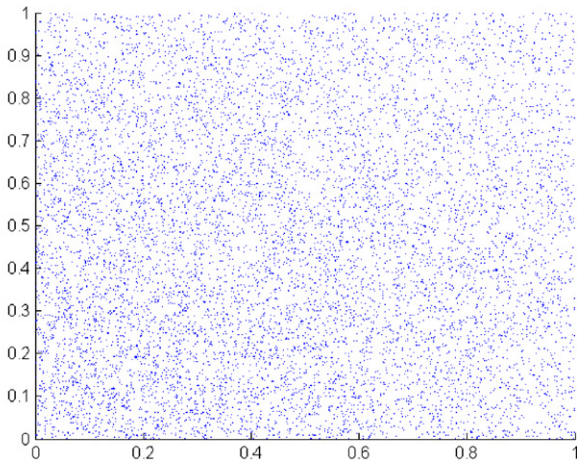


Fig. 2. The left picture shows the clustered model of home-points and the right shows its special case.

infinity. Thus, the per-node capacity of a network is the maximum arrival rate  $\lambda$  that the network can stably support [29].

### 3. Main results

In this paper, we provide two kinds of mobility cases. Under clustered model  $(m(n), r(n))$ , we define  $\zeta(n) = \sqrt{\log(m(n))/m(n)}$  as the critical transmission range [30] for connectivity when all nodes are static. The bounds on multicast capacity for two different mobility cases are given as follows. More details and explanations can be seen in Sections 4 and 5. Some mainly used notations are listed in Table 1.

#### 3.1. Strong mobility case $\zeta(n) = o\left(\frac{1}{\theta(n)}\right)$

The per-node multicast capacity is  $\left(\frac{1}{\sqrt{n_d}\theta(n)}\right)$ . Using an optimal scheduling scheme and routing policy, we have the lower bound on multicast capacity  $\lambda = \left(\frac{1}{\sqrt{n_d}\theta(n)}\right)$  under transmission range  $R_T = \left(\frac{1}{\sqrt{n}}\right)$ . That makes the lower bound asymptotically approaching to the upper bound on multicast capacity, i.e., the lower bound is tight. In this case, our scheme fully takes advantage of mobility and makes the concurrent scheduling as much as possible.

#### 3.2. Weak mobility case $\zeta(n) = \omega\left(\frac{1}{\theta(n)}\right)$

Using transmission range  $R_T = \left(\sqrt{\frac{\log m(n)}{m(n)}}\right)$ , we have the multicast throughput is

$$\lambda = \begin{cases} \left(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}}\right) & \text{when } n_d = O\left(\frac{m(n)}{\log m(n)}\right) \\ \left(\frac{1}{n}\right) & \text{when } n_d = \left(\frac{m(n)}{\log m(n)}\right) \end{cases}$$

In this case, we approximate it to the static case, using TDMA scheduling scheme to achieve the multicast throughput.

**Table 1**

Main notations used in this paper.

Notation	Meaning
$X_i(t)$	The position of node $i$ at time $t$
$d_{ij}(t)$	The distance between node $i$ and $j$ at time $t$
$X_i^h$	The home-point of mobile node $i$
$\gamma$	Activity exponent
$\theta(n)$	Activity coefficient
$m(n)$	The number of clusters
$r(n)$	The radius of a cluster
$n_s$	The number of source nodes in all multicast sessions
$n_d$	The number of destination nodes in one multicast session
$V_{i,D}$	The set of destination nodes of $i$ th multicast session
$\mathcal{X}_{i,D}$	The set of destination home-points of $i$ th multicast session
$A_{tes}$	An arbitrary tessellation element
$N_h(A_{tes})$	The number of home-points in the tessellation $A_{tes}$
$K$	$K^2$ -TDMA
$\lambda$	Per-node multicast capacity
$\mu^{S(i,j)}$	The probability link capacity between node $i$ and $j$ under the scheduling scheme $S$
$\zeta(n)$	Critical transmission range
$S^s$	Scheduling scheme under strong mobility case
$S^w$	Scheduling scheme under weak mobility case

From the results, we can see, besides the impact factor  $\sqrt{n_d}$ , the strong case is only concerned with  $\theta(n) = n^{\frac{1-\gamma}{2}}$ , while the weak case is just similar to the static situation, but having the result to do with the clustered model.

### 4. Multicast scheme

We define  $\zeta(n) = \sqrt{\log(m(n))/m(n)}$  in order to assure any two cluster centers can communicate with each other. Note that it is not to say all cluster centers can directly communicate.  $\zeta(n)$  just guarantees two cluster centers can contact each other under some relay schemes. See Fig. 3, the center in A wants to touch the center in C. Under  $\zeta(n) = \sqrt{\log(m(n))/m(n)}$ , this communication can be realized by using the center in B as a relay. In the static case,  $\zeta(n)$  denotes the minimal transmission range used to guarantee network connectivity.

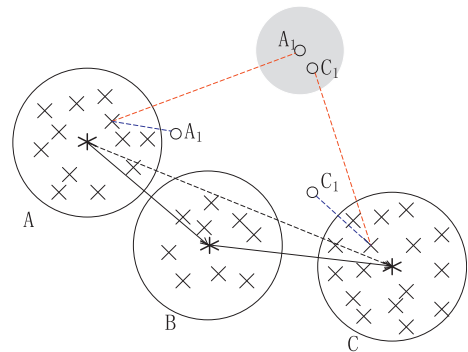
The denominator of Eq. (1), i.e.,  $\int_{\mathcal{O}} s(\theta(n) \|X - X_i^h\|) dX$ , is of order  $\frac{1}{\theta^2(n)}$  [24]. From this result, we can derive the node moving in an area of  $\left(\frac{1}{\theta^2(n)}\right)$  with high probability, i.e., the mobile radius is roughly limited to  $\left(\frac{1}{\theta(n)}\right)$ .

Using clustered model, we analyze the per-node multicast capacity under different node mobilities. We define two kinds of mobility cases through the following rules:

**Rule 1:** If the network and mobile nodes satisfy the condition  $\zeta(n) = o\left(\frac{1}{\theta(n)}\right)$ , we say the node mobility is strong, i.e., *strong mobility case*.

**Rule 2:** If the network and mobile nodes satisfy the condition  $\zeta(n) = \omega\left(\frac{1}{\theta(n)}\right)$ , we say the node mobility is weak, i.e., *weak mobility case*.

In Rule 1, we can obtain that the mobility exceeds the minimal transmission range. Thus mobility plays an important part in helping communication between two nodes and overcoming the disadvantage accompanied by clustering. On the contrary, in Rule 2, mobility is trivial. To some extent, we cannot look forward mobility to assist-



**Fig. 3.** The asterisk denotes the cluster center. The black solid line and the black dotted line mean a process of clustered centers communication by relay B. The red dash denotes the strong mobility case and the blue dash denotes the weak mobility case. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

ing us in exchanging information. See Fig. 3, when in strong impact on communication under strong and weak mobility cases mobility case, two nodes  $A_1, C_1$  can conquer clustering and directly communicate in the gray area; but when in weak case, two nodes cannot make use of mobility to directly exchange information.

**Lemma 1.** [24] Suppose that  $\{X_i^h, 1in\}$  are deployed on  $\mathcal{O}$  according to  $(m(n), r(n))$  Clustered model. The area of  $\mathcal{O}$  is divided by regular tessellations and each tessellation element has the area of  $|A_{tes}| (16 + \delta)\zeta^2(n)$ , for some small  $\delta > 0$ , and defined with  $N_h(A_{tes})$ , the number of home-points inside  $A_{tes}$ , then uniformly over the tessellations w.h.p.  $N_h(A_{tes})$  is comprised between  $\frac{n|A_{tes}|}{2}$  and  $2n|A_{tes}|$ , i.e.,  $\frac{n|A_{tes}|}{2} < \inf N_h(A_{tes})$   $\sup N_h(A_{tes}) < 2n|A_{tes}|$ .

#### 4.1. Scheduling scheme under strong mobility case

In this section, we will propose a proper scheduling scheme  $\mathcal{S}^s$  under strong mobility case. In previous studies, especially those using mobility to increase the overall capacity, the transmission range cannot be increased too large because it incurs much interference over possible concurrent transmissions. It is important to reduce the transmission range to an appropriate value which can both guarantee the network connectivity and maximize the overall capacity. In this paper, the mobility is different degrees and the mobile region is partial, but this problem also exists. Based on previous research, we choose  $R_T = \left(\frac{1}{\sqrt{n}}\right)$  as transmission range under strong mobility case. When two nodes move close to each other at a distance of  $\left(\frac{1}{\sqrt{n}}\right)$ , they can directly exchange data.

**Scheduling scheme  $\mathcal{S}^s$ :** [24] Given a network  $\mathcal{O}$  comprising  $n$  nodes moving on its surface, scheduling scheme  $\mathcal{S}^s$  enables transmission between node  $i$  and node  $j$  when the following conditions are satisfied:

$$d_{ij}(t) < R_T = \frac{c_1}{\sqrt{n}}$$

$$\min(d_{kj}(t), d_{ki}(t)) > (1+\Delta)R_T$$

This scheme is similar to the protocol model [1]. For every other node  $k$  in the network simultaneously transmitting,  $c_1$  is a constant and the quantity  $\Delta$  is a factor meaning a guard zone which prevents the simultaneous transmission in this guard area. Moreover, the transmission bandwidth is equally shared in two directions. It has been proved in [24] the scheduling scheme  $\mathcal{S}^s$  which use  $\left(\frac{1}{\sqrt{n}}\right)$  as transmission range is optimal.

#### 4.2. Scheduling scheme under weak mobility case

Under weak mobility case, we have  $\zeta(n) = \omega\left(\frac{1}{\theta(n)}\right)$ . Therefore, mobility does not play an important role in increasing the capacity. We can regard this case as static approximately. We must choose  $R_T = \Theta(\zeta(n))$  to guarantee the network connectivity.

**Scheduling scheme  $\mathcal{S}^w$ :** We use  $K^2$ -TDMA scheduling scheme based on a tessellation partition which is similar to Lemma 1, see Fig. 4, with side length is  $\sqrt{(16 + \delta)\zeta(n)}$ .

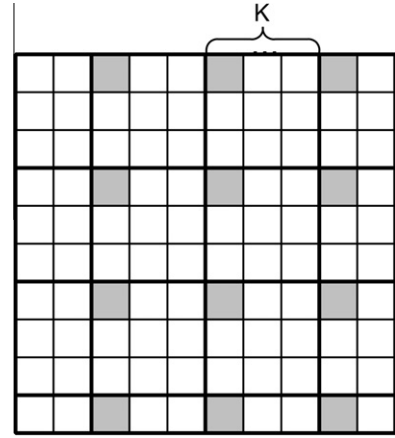


Fig. 4.  $K^2$ -TDMA. The gray tessellations denote concurrent scheduling.

**Lemma 2.** Under physical model, for any threshold value  $\beta$ , there exists a constant  $K, 0 < K < \infty$ , which can guarantee each tessellation to be successfully scheduled in  $K^2$  slots, i.e., making the capacity of each tessellation is  $\Theta(1)$ .

**Proof.** Due to the adjacent tessellations communication, we obtain:

$$\|X_i^h - X_j^h\| \sqrt{5} \sqrt{(16 + \delta)\zeta(n)}.$$

Then, according to Eq. (2), we have:

$$P \cdot \|X_i^h - X_j^h\|^{-\alpha} P \cdot (\sqrt{5} \sqrt{(16 + \delta)\zeta(n)})^{-\alpha}. \quad (3)$$

Considering a  $K^2$ -TDMA, we have the following inequality:

$$\sum_{k=1}^{\infty} \|X_k^h - X_j^h\|^{-\alpha} \sum_{i=1}^{\infty} 8i \cdot [(Ki - 2)\sqrt{(16 + \delta)\zeta(n)}]^{-\alpha}$$

$$N_0 + \sum_{k=1}^{\infty} P \|X_k^h - X_j^h\|^{-\alpha} N_0 + \sum_{i=1}^{\infty} P \cdot 8i \cdot [(Ki - 2)\sqrt{(16 + \delta)\zeta(n)}]^{-\alpha} \quad (4)$$

When  $\alpha > 2$ , the series

$$\sum_{i=1}^{\infty} \frac{P \cdot 8i}{[(Ki - 2)^\alpha]} \text{converges.}$$

According to (2), we combine (3) and (4), then having:

$$\text{SINR} \frac{P \cdot 5^{-\frac{\alpha}{2}} \cdot (\sqrt{(16 + \delta)\zeta(n)})^{-\alpha}}{N_0 + P \cdot (\sqrt{(16 + \delta)\zeta(n)})^{-\alpha} \cdot \sum_{i=1}^{\infty} \frac{8i}{(Ki - 2)^\alpha}}.$$

We denote  $G(n) = \sqrt{(16 + \delta)\zeta(n)}$ , then

$$\text{SINR} - \beta(G(n))^{-\alpha} \left[ \frac{P \left( 5^{-\frac{\alpha}{2}} - \beta \cdot \sum_{i=1}^n \frac{8i}{(Ki - 2)^\alpha} \right) - \beta N_0 G(n)^\alpha}{N_0 G(n)^\alpha + P \sum_{i=1}^{\infty} \frac{8i}{(Ki - 2)^\alpha}} \right]$$

For any given  $\beta$ , there exists a large enough constant  $0 < K < \infty$  such that

$$5^{-\frac{\alpha}{2}} - \beta \cdot \sum_{i=1}^n \frac{8i}{(Ki - 2)^\alpha} > 0.$$

Thus, we get  $\text{SINR} > \beta$ , which completes the proof.  $\square$

## 5. Multicast capacity analysis

### 5.1. Routing Policy $\mathcal{R}$

We consider a multicast session  $i \rightarrow \mathcal{V}_{i,D}$ , thus having home-points  $X_i^h \rightarrow \mathcal{X}_{i,D}^h$  correspondingly. Based on the following routing policy  $\mathcal{R}$  described in Algorithm 1, we construct a multicast routing tree for each multicast session. The algorithm is similar to [31].

In Algorithm 1, under strong mobility case, we consider the side length of tessellation is  $\frac{c}{\theta(n)}$ , where  $c$  is a constant. First, the side length can satisfy the condition of Lemma 1 which guarantees each tessellation has home-points. Second, the side length is equal to the mobile radius being order of  $\left(\frac{1}{\theta(n)}\right)$ . It can make nodes meet each other with high probability in two adjacent tessellations under scheduling scheme  $\mathcal{S}^t$ .

Similarly, under scheduling scheme  $\mathcal{S}^s$ , the side length of the tessellation in Algorithm 1 is  $\sqrt{(16 + \delta)\zeta(n)}$ .

**Algorithm 1.** Multicast routing based on Manhattan routing

- 
- 1: Partition the area into a sequence of regular tessellations, using the method described in Lemma 1. Each tessellation with side length  $\frac{c}{\theta(n)}$  (If under the weak mobility case, the side length will be  $\sqrt{(16 + \delta)\zeta(n)}$ ), illustrated in Fig. 5. Thus, it could guarantee each tessellation has home-points in it, denoted by  $(i,j)$  when it is the  $i$ th column and  $j$ th row.
  - 2: Select independently and randomly  $n_d$  home-points formed a destination set, denoted by  $\mathcal{X}_{i,D}^h = \{X_{i1}^h, X_{i2}^h, \dots, X_{in_d}^h\}$ .
  - 3: Build an Euclidean spanning tree denoted as EST( $X$ ) according to Algorithm 2.
  - 4: For each link  $uv$  in the tree EST( $X_i^h$ ), assume that  $u$  and  $v$  are inside tessellation  $(i_u, j_u)$  and tessellation  $(i_v, j_v)$  respectively. Find a home-point  $w$  in tessellation  $(i_u, j_u)$  (or tessellation  $(i_u, j_v)$ ), i.e.,  $uwv$  is a Manhattan path connecting  $u$  and  $v$ . The resulted structure by uniting all such paths for all links in EST( $X_i^h$ ) will serve the routing guideline for multicast.
  - 5: For each edge  $uw$  in EST( $X_i^h$ ), find a home-point in each of the tessellation that are crossed by line  $uw$ . Connect these home-points in sequence to form a path, denoted as  $X(u, v)$ , connecting points  $u$  and  $v$ . Notice that here such structure may not be a tree. If this is the case, remove the cycles that do not contain home-points from  $\mathcal{X}_{i,D}^h$ . Denote the resulted tree as MT( $X_i^h$ ).
  - 6: Link all the associated the mobile nodes to construct a final node tree called MTR( $v_i$ ) according to above MT( $X_i^h$ ), due to each home-point has a mobile node moving around it.
- 

### 5.2. Lower bound on multicast capacity under strong mobility case

Probability link capacity  $\mu^S(i, j)$ :

$$\mu^S(i, j) = E[1_{(i,j) \in \pi^S(t)} | \mathcal{F}_{ij}],$$

where  $\pi^S(t)$  is a selected set of node pairs which can simultaneous transmit at time  $t$  under a stationary ergodic scheduling scheme  $\mathcal{S}$ .  $\mathcal{F}_{ij}$  is the Borel-field generated by  $\{X_i^h, X_j^h\}$ . Probability link capacity is maximal traffic flow between node  $i$  and node  $j$  [25].

**Lemma 3.** [24] In strong mobility case  $\zeta(n) = o\left(\frac{1}{\theta(n)}\right)$ , under the scheduling scheme  $\mathcal{S}^s$ , for any pair of nodes  $(i, j)$  and any finite  $c_1 > 0$ , we have the probability link capacity

$$\mu^{\mathcal{S}^s}(i, j) = \left( \Pr \left\{ d_{ij} \frac{c_1}{\sqrt{n}} \middle| \mathcal{F}_{ij} \right\} \right).$$

From the derivation in the literature [24], we obtain:

$$\mu^{\mathcal{S}^s}(i, j) = \left( g(n) \eta(\theta(n)) \left\| X_j^h - X_i^h \right\| \right), \quad (5)$$

where  $g(n) = \pi c_1^2 \frac{\theta^2(n)}{n}$  and  $\eta(\|Y\|) = \int_{X \in \mathbb{R}} s(\|X - Y\|) s(\|X\|) dX$ .

**Lemma 4.** Given a tessellation  $A_{tes}$ , the probability of a multicast flow going through the  $A_{tes}$  is  $\min\left(\frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)}, 1\right)$  in strong mobility case.

**Proof.** We denote  $l$  as the length of a multicast flow,  $l_h$  and  $l_v$  denote the horizontal and vertical projects respectively. In strong mobility case, the side length of  $A_{tes}$  is  $\frac{c}{\theta(n)}$ .  $\Pr(l, A_{tes})$  denotes the probability of a multicast flow going through the  $A_{tes}$ . Then we have,

$$\Pr(l, A_{tes}) = \frac{c^2}{\theta^2(n)} \left( \frac{l_h + l}{\frac{c}{\theta(n)}} + 1 \right) = \frac{c(l_h + l)}{\theta(n)} + \frac{c^2}{\theta^2(n)} \quad (6)$$

By Lemma 8 of [31], the length of EST is  $2\sqrt{2}\sqrt{n_d}$ , then we get that

$$\frac{c(l_h + l)}{\theta(n)} \frac{\sqrt{2}lc}{\theta(n)} \frac{4\sqrt{n_d}c}{\theta(n)}.$$

Finally, we derive:

$$\Pr(l, A_{tes}) = \min \left( \frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)}, 1 \right).$$

With all sessions,

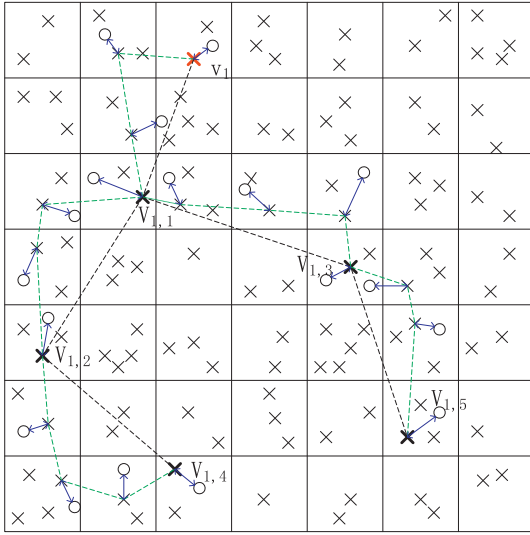
$$\Pr^{all}(l, A_{tes}) = O \left( n_s \cdot \left( \frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)} \right) \right).$$

By Lemma 7 of [31], we have

$$\Pr^{all}(l, A_{tes}) 2n_s \cdot \left( \frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)} \right),$$

which completes the proof.  $\square$

Here we show how to use routing policy  $\mathcal{R}$  and scheduling scheme  $\mathcal{S}^s$  to gain a lower bound. Assuming there exists a multicast session  $v_1 \rightarrow \{v_{1,1}, v_{1,2}, \dots, v_{1,k}\}$ . According to



Manhattan Multicast Tree

**Fig. 5.** The figure illustrates a construction of Manhattan multicast tree. The red bold cross denotes a source, the black bold crosses denote  $n_d$  destinations, the common crosses use as relays in the Manhattan path, and circles denote mobile nodes. The dash lines mark an original spanning tree of home-points, the green dotted lines mark a Manhattan multicast tree of home-points. Last, make the mobile nodes associated with their home-points, shaping a Manhattan multicast tree of nodes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

their home-points, we use routing policy  $\mathcal{R}$  to construct a Manhattan multicast tree. The real transmission process is that the home-points of adjacent tessellations use scheduling scheme  $\mathcal{S}^s$  to make the associated nodes transmit data. For example, in Fig. 5, source node  $v_1$  transmits data to one of its destination nodes  $v_{1,1}$ . According to Manhattan path, they first schedule home-points horizontally, and then vertically.

**Theorem 1.** Given a network  $\mathcal{O}$  with  $n$  mobile nodes, in strong mobility case, under scheduling scheme  $\mathcal{S}^s$ , the lower bound on per-node capacity is  $\lambda = \left(\frac{1}{\sqrt{n_d}\theta(n)}\right)$ .

**Proof.** Assume  $A_{tes}$  and  $B_{tes}$  are adjacent tessellations. Let  $\underline{N}_h(A_{tes})$  and  $\underline{N}_h(B_{tes})$  respectively be the lower bound of the number of mobile nodes whose home-points fall in  $A_{tes}$  and  $B_{tes}$ .

Since  $\zeta(n) = o\left(\frac{1}{\theta(n)}\right)$ , thanks to Lemma 1, we have  $\underline{N}_h(A_{tes}) = \underline{N}_h(B_{tes}) = \frac{c^2 n}{2\theta^2(n)}$ .

Combining Lemma 3, we have the feasible maximal traffic flow between two adjacent tessellations is

$$\mu^{\mathcal{S}^s}(\bar{d}_{A_{tes}, B_{tes}}) \cdot \underline{N}_h(A_{tes}) \underline{N}_h(B_{tes}).$$

Because  $\bar{d}_{A_{tes}, B_{tes}} = \frac{\sqrt{5}c}{\theta(n)}$ , we unite (5), then having

$$\mu^{\mathcal{S}^s}(\bar{d}_{A_{tes}, B_{tes}}) = g(n)\eta(\sqrt{5}c),$$

where  $c$  must be appropriately chosen to make  $\eta(\sqrt{5}c) > 0$ .

From Lemma 4, we know the maximal load is

$$O\left(n_s \cdot \left(\frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)}\right)\right).$$

Then we have the lower bound of strong mobility case is

$$\lambda \frac{\mu^{\mathcal{S}^s}(\bar{d}_{A_{tes}, B_{tes}}) \cdot \underline{N}_h(A_{tes}) \underline{N}_h(B_{tes})}{n_s \cdot \left(\frac{4c\sqrt{n_d}}{\theta(n)} + \frac{2n_d c^2}{\theta^2(n)}\right)}. \quad (7)$$

Deriving from (7), we finally obtain:

$$\lambda = \left(\frac{1}{\sqrt{n_d}\theta(n)}\right).$$

### 5.3. Lower bound on multicast capacity under weak mobility case

**Lemma 5.** Given a tessellation  $A_{tes}$ , the probability of a multicast flow going through the  $A_{tes}$  is  $\min(4\sqrt{(16 + \delta)}\zeta(n)\sqrt{n_d} + 2n_d(16 + \delta)\zeta^2(n), 1)$  in weak mobility case.

The proof is similar to Lemma 4.

**Theorem 2.** Given a network  $\mathcal{O}$  with  $n$  mobile nodes, in weak mobility case, under scheduling scheme  $\mathcal{S}^s$ , the lower bound on per-node capacity is

$$\lambda = \begin{cases} \left(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}}\right) & \text{when } n_d = O\left(\frac{m(n)}{\log m(n)}\right) \\ \left(\frac{1}{n}\right) & \text{when } n_d = \left(\frac{m(n)}{\log m(n)}\right) \end{cases}$$

**Proof.** Assume  $A_{tes}$  and  $B_{tes}$  are adjacent tessellations. Since  $\zeta(n) = \omega\left(\frac{1}{\theta(n)}\right)$ , thanks to the scheduling scheme  $\mathcal{S}^s$  using  $K^2$ -TDMA, from Lemma 5, we know the maximal load is

$$O(n_s \cdot (4\sqrt{(16 + \delta)}\zeta(n)\sqrt{n_d} + 2n_d(16 + \delta)\zeta^2(n))).$$

Thus, we have the lower bound of weak mobility case is

$$\lambda \frac{\frac{1}{K^2}}{n_s \cdot (4\sqrt{(16 + \delta)}\zeta(n)\sqrt{n_d} + 2n_d(16 + \delta)\zeta^2(n))}. \quad (8)$$

Deriving from (8) and combining the broadcast result in [10], we finally obtain:

$$\lambda = \begin{cases} \left(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}}\right) & \text{when } n_d = O\left(\frac{m(n)}{\log m(n)}\right) \\ \left(\frac{1}{n}\right) & \text{when } n_d = \left(\frac{m(n)}{\log m(n)}\right) \end{cases}$$

### 5.4. Upper bound on multicast capacity under strong mobility case

In [24], the upper bound on unicast is gained by solving a Maximum Concurrent Flow problem over GRGG. A network  $\mathcal{O}$  is divided into two regions  $I_{\mathcal{L}}$  and  $E_{\mathcal{L}}$  by using an arbitrary simple, regular and closed curve  $\mathcal{L}$ . The upper bound on per-node capacity is obtained by:

$$\lambda \frac{\sum_{i: X_i^h \in I_{\mathcal{L}}} \sum_{j: X_j^h \in E_{\mathcal{L}}} \mu_{ij}}{\sum_{s: X_s^h \in I_{\mathcal{L}}} \sum_{d: X_d^h \in E_{\mathcal{L}}} \lambda_{sd}}.$$

The numerator of above inequality means the maximum entire traffic that crosses the  $\mathcal{L}$ , while

the denominator is the number of traffic flows passing through  $\mathcal{L}$ .

In this paper, we also use the similar method to prove our upper bound.

**Lemma 6.** [24] *Under the assumption  $\int x^3 \eta(x) dx < \infty$ , for any convex, simple, regular, closed curve  $\mathcal{L}$ :*

$$\theta^2(n) \int_{X \in \mathcal{L}} \int_{Y \in E_{\mathcal{L}}} \eta(\theta(n) \|X - Y\|) d_X d_Y = \left( \frac{1}{\theta(n)} \right)$$

**Theorem 3.** *Given a network  $\mathcal{O}$  with  $n$  mobile nodes, in strong mobility case, the upper bound on per-node capacity is  $\lambda = \left( \frac{1}{\sqrt{n_d} \theta(n)} \right)$ .*

**Proof.** We denote  $l_{k,i}$  as the side of  $i$ th edge of  $k$ th multicast session. As the area of network is normalized to 1, the probability that  $l_{k,i}$  will pass through  $\mathcal{L}$  is  $l_{k,i} \cos \psi_{k,i}$ , being  $\psi_{k,i}$  the horizontal angle of  $l_{k,i}$ .

A random variable  $\varepsilon_{k,i}$  is defined as follows:

$$\varepsilon_{k,i} = \begin{cases} 1 & \text{when } l_{k,i} \text{ crossing } \mathcal{L} \\ 0 & \text{when } l_{k,i} \text{ not crossing } \mathcal{L} \end{cases}$$

The number of multicast flows crossing  $\mathcal{L}$  is denoted by  $F_{\mathcal{L}}$ , then

$$F_{\mathcal{L}} = E \left( \sum_{k=1}^{n_s} \sum_{i=1}^{n_d} \varepsilon_{k,i} \right) = \sum_{k=1}^{n_s} \sum_{i=1}^{n_d} E(\varepsilon_{k,i}) = \sum_{k=1}^{n_s} \sum_{i=1}^{n_d} l_{k,i} \cos \psi_{k,i}$$

By Lemma 9 of [10], we have:

$$\sum_{k=1}^{n_s} \sum_{i=1}^{n_d} l_{k,i} \cos \psi_{k,i} \leq c_2 n_s \sqrt{n_d},$$

being  $c_2$  a constant.

Thus, we have  $F_{\mathcal{L}} \leq c_2 n_s \sqrt{n_d}$ .

According to the result in [24], we have:

$$\sum_{i: X_i^h \in \mathcal{L}} \sum_{j: X_j^h \in E_{\mathcal{L}}} \mu_{ij} n^2 g(n) \int_{X \in \mathcal{L}} \int_{Y \in E_{\mathcal{L}}} \eta(\theta(n) \|X - Y\|) d_X d_Y$$

We obtain the upper bound

$$\lambda \frac{\sum_{i: X_i^h \in \mathcal{L}} \sum_{j: X_j^h \in E_{\mathcal{L}}} \mu_{ij} n^2 g(n) \int_{X \in \mathcal{L}} \int_{Y \in E_{\mathcal{L}}} \eta(\theta(n) \|X - Y\|) d_X d_Y}{F_{\mathcal{L}}} \leq \frac{1}{c_2 n_s \sqrt{n_d}}$$

Thanks to Lemma 6, finally, we have  $\lambda = \left( \frac{1}{\sqrt{n_d} \theta(n)} \right)$ .  $\square$

## 6. Literature review

We review some existing works on the capacity scaling laws and the development of mobility models of wireless ad hoc networks. While, we present the differences between our work and some related ones.

Gupta and Kumar [1] studied the unicast capacities for protocol and physical interference models. They showed that the per-node throughput is of order  $\left( \frac{1}{\sqrt{n \log n}} \right)$ . Then, for Gaussian channel model, Franceschetti et al. [7] showed that the unicast capacity is  $\left( \frac{1}{\sqrt{n}} \right)$  in the network of ran-

domly located nodes. Keshavarz-Haddad et al. [2] studied the broadcast capacity, and the per session broadcast capacity is of order  $\Theta(1/n)$ . Shakkottai et al. [5] studied the multicast capacity, i.e., more generalized dissemination session, they designed a *comb routing scheme*, by which the per session multicast throughput is of order  $\left( \frac{1}{\sqrt{n_d}} \right)$ , where  $n_d$  is the number of destinations. Li [31], for random networks, assuming that  $n_s = \left( \log n_d \sqrt{n \log n / n_d} \right)$ , the per session capacity of  $n_s$  multicast sessions is of order  $\left( 1 / \sqrt{n_d n \log n} \right)$  when  $n_d = O(n \log n)$ , and is of order  $\Theta(1/n)$  when  $n_d = \Omega(n \log n)$ . Liu et al. [6] studied a data gathering communication model in which a subset of nodes send data to some designated destination nodes, while other nodes serve as relays. Their results show that the data gathering capacity is restricted by different factors and presents distinct scaling behaviors in the different scaling regimes of the number of source and destination nodes.

A *store-carry-forward* scheme goes further in using mobility to improve the capacity. Grossglauser and Tse [15] showed that mobility can help to increase the unicast capacity if allowing large delay. The average long-term throughput per-node can be kept constant even if the number of nodes increases. After that, several mobility model assumptions for network performance analysis have been proposed. For example, i.i.d model, random walk mobility model [16], Brownian mobility model [17], random way-point mobility model [18,19], and some restricted/local mobility models. Here, we refer to [32–34] for overview of state-of-the-art restricted mobility models.

In [32], Lozano et al. divided the network area into overlapping neighborhoods while  $n$  mobile nodes were limited to move within their assigned neighborhood. The result is concerned with node locations and neighborhood dimensions. When randomly locating nodes with  $n^\alpha$  neighborhoods,  $0 < \alpha < 1$ , a throughput of  $(n^{1-\alpha})$  can be achieved. This result can encompass both [1,15] as extreme situations. In [33], Mammen and Shah studied the maximal throughput scaling and the corresponding delay scaling in a random mobile network with restricted node mobility. The result is  $D(n) = \Theta(n \log n)$ , which is the same as the delay scaling without any mobility restriction. It demonstrates the restricted mobility does not affect delay scaling. In [34], Garetto and Leonardi worked on the asymptotic delay-throughput tradeoffs in mobile ad hoc networks comprising heterogeneous nodes with restricted mobility. By defining a power law of exponent  $\delta$ , authors analyzed delay-throughput tradeoffs under all possible values of  $\delta$ . In particular, when  $\delta = 2$ , it is possible to achieve almost constant delay and almost constant per-node throughput.

Our work differs from previous ones. Garetto et al. [24] showed the per-node capacity is  $\left( \frac{1}{\sqrt{n}} \right)$  when the mobility is limited to radius  $\frac{1}{\sqrt{n}}$  in a network with unit size. Then, Huang et al. [25] extended the work by using infrastructure support, which per-node capacity  $\left( \frac{1}{\sqrt{n}} \right) + (\min(k^2 c/n, k/n))$  is gained under strong mobility, and  $\Theta(\min(k^2 c/n, k/n))$  in other cases. The above two works consider the unicast capacity. In [35], Peng et al. studied the heterogeneity increasing multicast capacity, but it is in the static network



and the heterogeneity is just about the aspect of cluster. In [36], delay-throughput performance of mobile ad hoc networks was analyzed, but the heterogeneity in it is just about nodes and authors only give the delay-throughput on unicast. Our work gives the multicast capacity scaling for inhomogeneous mobile ad hoc networks.

## 7. Conclusion

In this paper, we conduct an analysis on upper bound and lower bound on multicast capacity for inhomogeneous mobile ad hoc networks. We develop and extend the mobility model by introducing two elements. One is through activity exponent  $\gamma$  to feature the mobility pattern, the other is through clustering parameters  $(m(n), r(n))$  to characterize the spatial inhomogeneities of nodes density. Taking advantage of different scheduling schemes and routing policies, our methodology derives that per-node multicast capacity is  $(\frac{1}{\sqrt{n_d} \theta(n)})$  in strong mobility case. While, in weak mobility case, when  $n_d = O(\frac{m(n)}{\log m(n)})$ , we have the multicast throughput is  $(\frac{1}{\sqrt{n_d}} \sqrt{\frac{m(n)}{n^2 \log m(n)}})$ ; when  $n_d = (\frac{m(n)}{\log m(n)})$ , the multicast throughput is  $(\frac{1}{n})$ . Our work provides deeper understanding of real mobility model and obtains the bounds of multicast capacity which can guide us in designing the mobile network.

There are several questions left for study. First, our work is applied in delay tolerant network. We could make some research on the problems of delay constraints or delay capacity tradeoffs. Second, we only concern with the inhomogeneous spatial distribution of nodes in mobile ad hoc network, but the heterogeneity of mobile nodes is not studied. Third, our paper will study a new model or improve our model which can apply to the roads and vehicular networks in some realistic scenarios.

## Acknowledgment

This work is supported by National Basic Research Program of China (No. 2010CB328101) and National Natural Science Foundation of China (No. 90818023).

## Appendix A

### Algorithm 2. Construction of EST

**Input:** the source home-point  $X_i^h$  and the set of its destination home-points  $\mathcal{X}_{i,D}^h$

**Output:** An Euclidean spanning tree EST ( $X$ ).

1: In the initial state, all home-points including  $X_i^h$  and set of its designation home-points  $\mathcal{X}_{i,D}^h$  are isolated, then there are  $n_d + 1$  connected components.

2: **for**  $i = 1:n_d$  **do**

3: Partition the deployment region into at most  $n_d + 1 - i$  square cells, each with side length

$$1/\lceil \sqrt{n_d + 1 - i} \rceil;$$

- 4: Find a cell that contains two home-points belonging to two different connected components. By connecting this pair of home-points, we merge the two connected components.
- 5: **end for**

**Lemma 7.** [[31]] Consider  $n$  independent random variables  $X_i \in \{0, 1\}$  with  $p = \Pr(X_i = 1)$ . Let  $X = \sum_{i=1}^n X_i$ . Then

$$\begin{cases} \Pr(X \leq \xi) e^{-\frac{2(np - \xi)^2}{n}}, & \text{when } 0 < \xi < n \cdot p \\ \Pr(X > \xi) < \frac{\xi(1-p)}{(\xi - np)^2}, & \text{when } \xi > n \cdot p \end{cases}$$

**Lemma 8.** [[31]] For any multicast session of  $n_d + 1$  nodes deployed in a network of  $a^2$ , let EST ( $X$ ) denote an Euclidean spanning tree (EST) constructed by Algorithm 2, it holds that  $k = 1, 2, \dots, n_s, \|\text{EST}(X)\| \leq 2\sqrt{2} \cdot \sqrt{n_d} \cdot a$ .

**Lemma 9.** [[10]] The total edge length of the EMST of  $n$  nodes randomly and uniformly distributed in a  $d$ -dimensional cube of side-length  $a$  is asymptotic to  $\tau(d) \cdot n^{\frac{d-1}{d}} \cdot a$ , where  $\tau(d)$  is a constant depending only on the dimension  $d$ .

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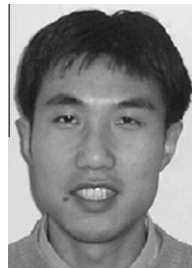
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